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PRODUCTION ESTIMATION

Inflow Performance Relationships for Solution-Gas Drive Wells

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Abstract

In calculating oilwell production, it has commonly been assumed that producing rates are proportional to drawdowns. Using this assumption, a well's behavior can be described by its productivity index (PI). This PI relationship was developed from Darcy's law for the steady-state radial flow of a single, incompressible fluid. Although Muskat pointed out that the relationship is not valid when both oil and gas flow in a reservoir, its use has continued for lack of better approximations. Gilbert proposed methods of well analysis utilizing a curve of producing rates plotted against bottom-hole well pressures; he termed this complete graph the inflow performance relationship (IPR) of a well.

The calculations necessary to compute IPR's from two-phase flow theory were extremely tedious before advent of the computer. Using machine computations, IPR curves were calculated for wells producing from several fictitious solution-gas drive reservoirs that covered a wide range of oil PVT properties and reservoir relative permeability characteristics. Wells with hydraulic fractures were also included. From these curves, a reference IPR curve was developed that is simple to apply and, it is believed, can be used for most solution-gas drive reservoirs to provide more accurate calculations for oilwell productivity than can be secured with PI methods. Field verification is needed.

Introduction

In calculating the productivity of oil wells, it is commonly assumed that inflow into a well is directly proportional to the pressure differential between the reservoir and the wellbore — that production is directly proportional to drawdown. The constant of proportionality is the PI, derived from Darcy's law for the steady-state radial flow of a single, incompressible fluid. For cases in which this relationship holds, a plot of the producing rates vs the corresponding bottom-hole pressures results in a straight line (Fig. 1). The PI of the well is the inverse of the slope of the straight line.

However, Muskat[1] pointed out that when two-phase liquid and gas flow exists in a reservoir, this relationship should not be expected to hold; he presented theoretical calculations to show that graphs of producing rates vs bottom-hole pressures for two-phase flow resulted in curved rather than straight lines. When curvature exists, a well cannot be said to have a single PI because the value of the slope varies continuously with the variation in drawdown. For this reason, Gilbert[2] proposed methods of well analysis that could utilize the whole curve of producing rates plotted against intake pressures. He termed this complete graph the inflow performance relationship (IPR) of a well.

Although the straight-line approximation is known to have limitations when applied to two-phase flow in the reservoir, it still is used primarily because no simple substitutes have been available. The calculations necessary to compute IPR's from two-phase flow theory have been extremely tedious. However, recently the approximations of Weller[3] for a solution-gas drive reservoir were programmed for computers. The solution involved the following simplifying assumptions: (1) the reservoir is circular and completely bounded with a completely penetrating well at its center; (2) the porous medium is uniform and isotropic with a constant water saturation at all points; (3) gravity effects can be neglected; (4) compressibility of rock and water can be neglected; (5) the composition and equilibrium are constant for oil and gas; (6) the same pressure exists in both the oil and gas phases; and (7) the semisteady-state assumption that the tank-oil desaturation rate is the same at all points at a given instant. Weller's solution did not require the constant-GOR assumption.

The resulting computer program proved convenient to use and gave results closely approaching those furnished by the more complicated method of West, Garvin and Sheldon.[4] The program also includes the unique feature

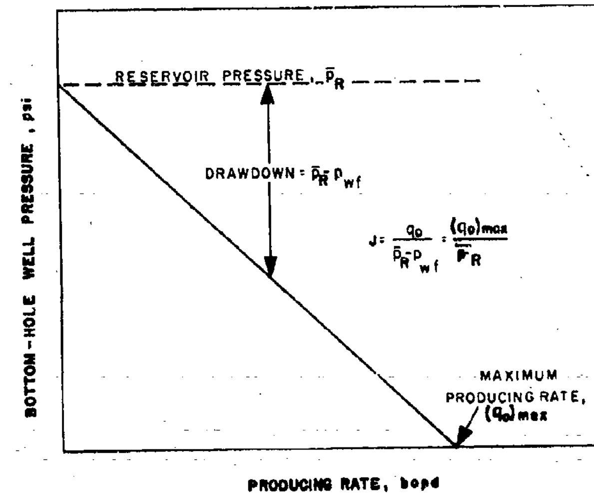

Fig. 1—*Straight-line inflow performance relationship.*

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[1]References given at end of paper.

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of making complete IPR predictions for a reservoir. Such predictions for a typical solution-gas drive reservoir are shown as a family of IPR curves on Fig. 2. Note that they confirm the existence of curvature.

It appeared that if several solution-gas drive reservoirs were examined with the aid of this program, empirical relationships might be established that would apply to solution-gas drive reservoirs in general. This paper summarizes the results of such a study that dealt with several simulated reservoirs covering a wide range of conditions. These conditions included differing crude oil characteristics and differing reservoir relative permeability characteristics, as well as the effects of well spacing, fracturing and skin restrictions.

The investigation sought relationships valid only below the bubble point. Computations were made for reservoirs initially above the bubble point, but only to ensure that this initial condition did not cause a significant change in behavior below the bubble point.

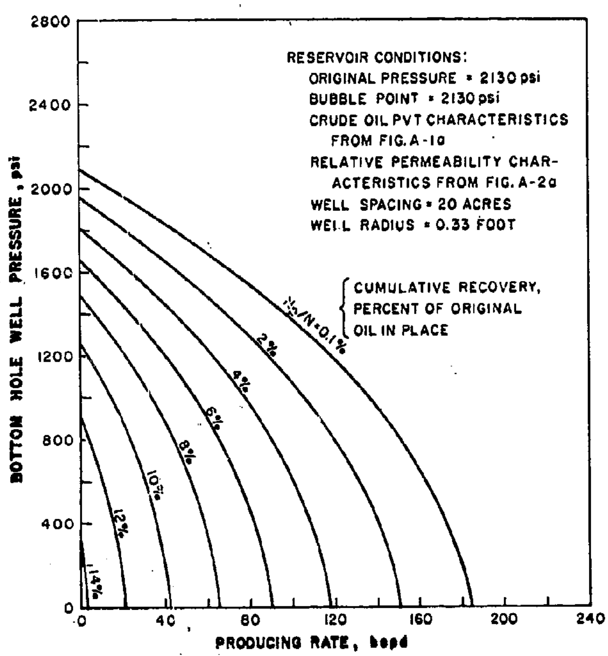

Fig. 2—*Computer-calculated inflow performance relationships for a solution-gas drive reservoir.*

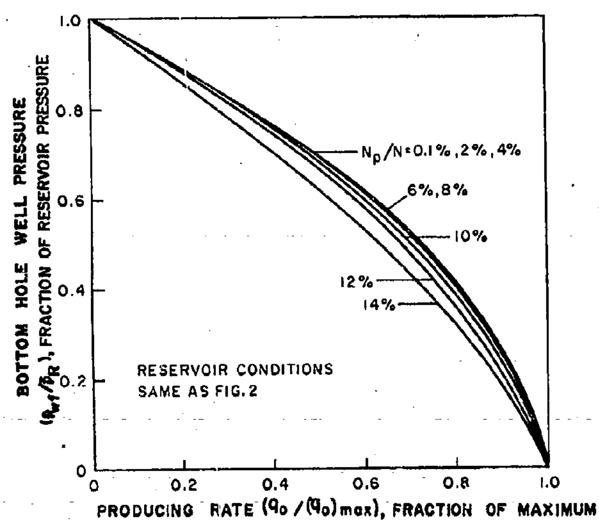

Fig. 3—*Dimensionless inflow performance relationships for a solution-gas drive reservoir.*

Shape of Inflow Performance Relationship Curves with Normal Deterioration

As depletion proceeds in a solution-gas drive reservoir, the productivity of a typical well decreases, primarily because the reservoir pressure is reduced and because increasing gas saturation causes greater resistance to oil flow. The result is a progressive deterioration of the IPR's, typified by the IPR curves in Fig. 2. Examination of these curves does not make it apparent whether they have any properties in common other than that they are all concave to the origin.

One useful operation is to plot all the IPR's as "dimensionless IPR's". The pressure for each point on an IPR curve is divided by the maximum or shut-in pressure for that particular curve, and the corresponding production rate is divided by the maximum (100 percent drawdown) producing rate for the same curve. When this is done, the curves from Fig. 2 can be replotted as shown in Fig. 3. It is then readily apparent that with this construction the curves are remarkably similar throughout most of the producing life of the reservoir

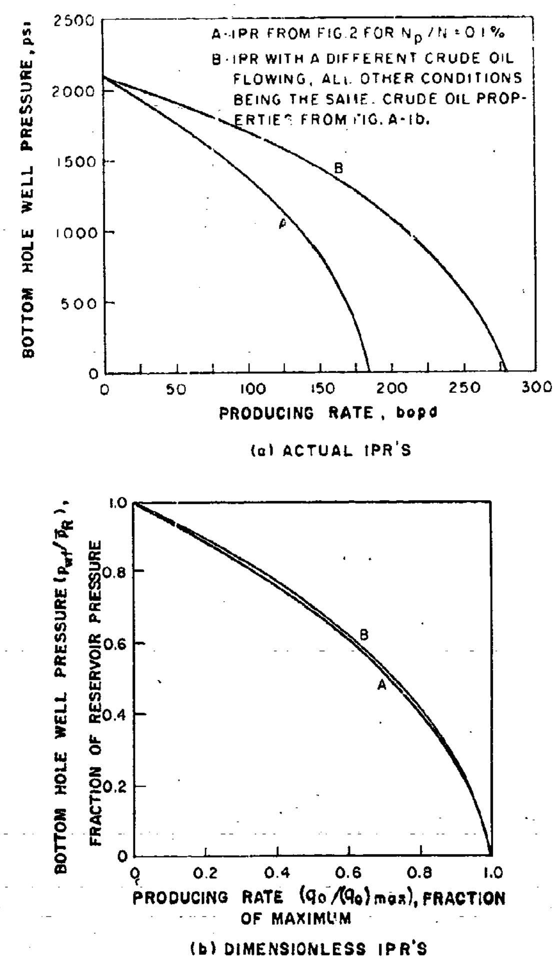

Fig. 4—*Effect of crude oil properties on IPR's.*

Effect of Crude Oil Characteristics On IPR Curves

From the foregoing results it appears that IPR curves differing over the life of a given reservoir actually possess a common relationship. To determine whether this same relationship would be valid for other reservoirs, IPR calculations were made on the computer for different conditions. The first run utilized the same relative permeabilities but a completely different crude oil. The new characteristics included a viscosity about half that of the first and a solution GOR about twice as great.

Fig. 4a compares the initial IPR's ($N_p/N = 0.1$ percent) for the two cases. As would be expected, with a less viscous crude (Curve B) the productivity was much greater than in the first case (Curve A). However, when plotted on a dimensionless basis (Fig. 4b) the IPR's are quite similar. As IPR's for the second case deteriorated with depletion, no greater change of shape occurred than was noted in the previous section. These two crude oils had about the same bubble point. IPR's were then calculated for a third crude oil with a higher bubble point. Again, the characteristic shape was noted.

Two further runs were made to explore the relationship under more extreme conditions. One utilized a more viscous crude (3-cp minimum compared with 1-cp minimum), and the other used a crude with a low solution GOR (300 scf/STB). With the more viscous crude, some straightening of the IPR's was noted. The low-GOR crude exhibited the same curvature noted in previous cases.

Runs were also made with the initial reservoir pressure exceeding the bubble point. During the period while the reservoir pressure was above the bubble point, the slopes of the IPR curves were discontinuous with the upper part being a straight line until the well pressure was reduced below the bubble point. Below this point the IPR showed curvature similar to that noted previously. After the reservoir pressure went below the bubble point, all the dimensionless IPR curves agreed well with the previous curves.

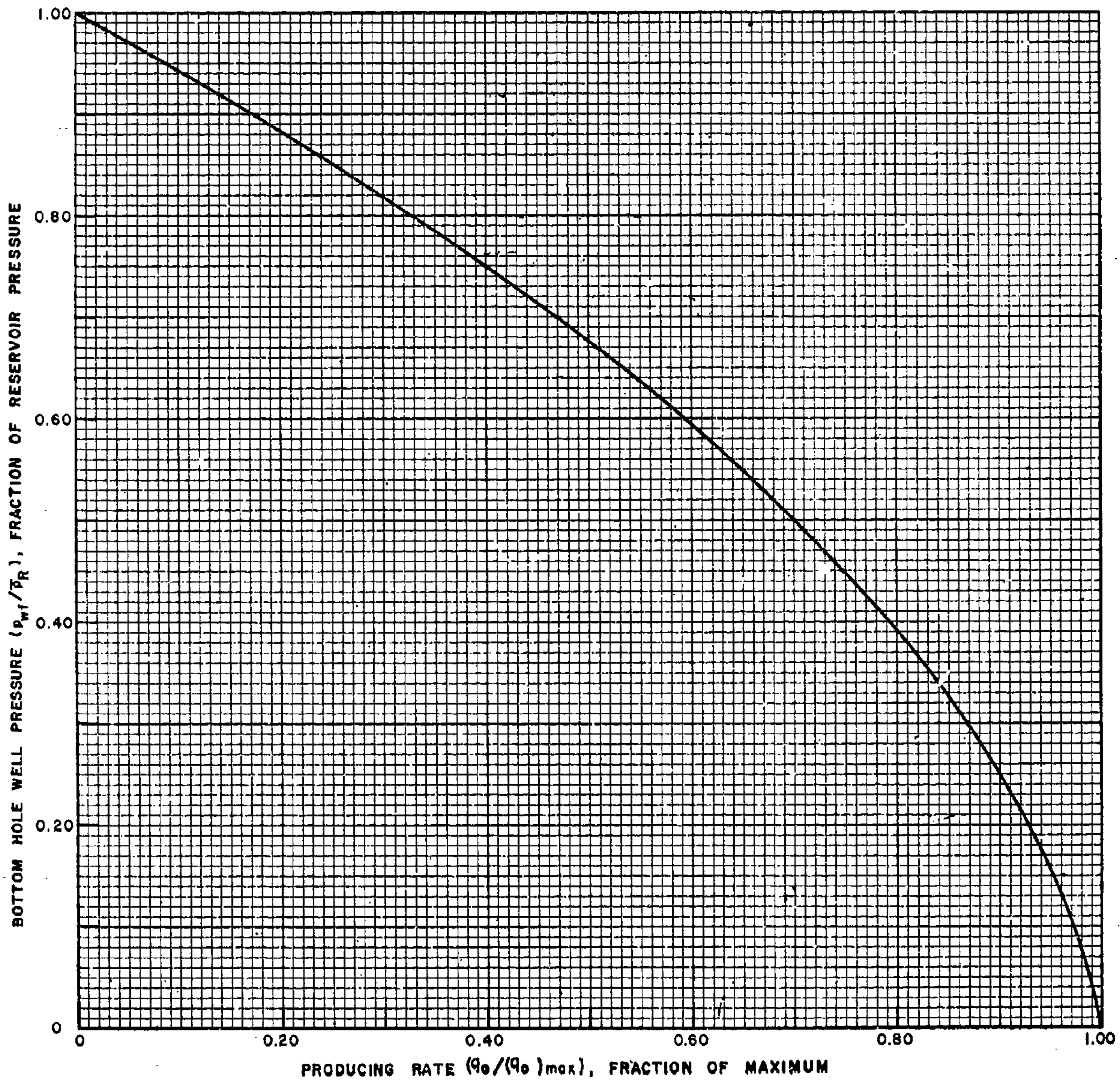

Fig. 5—*Inflow performance relationship for solution-gas drive reservoirs.*

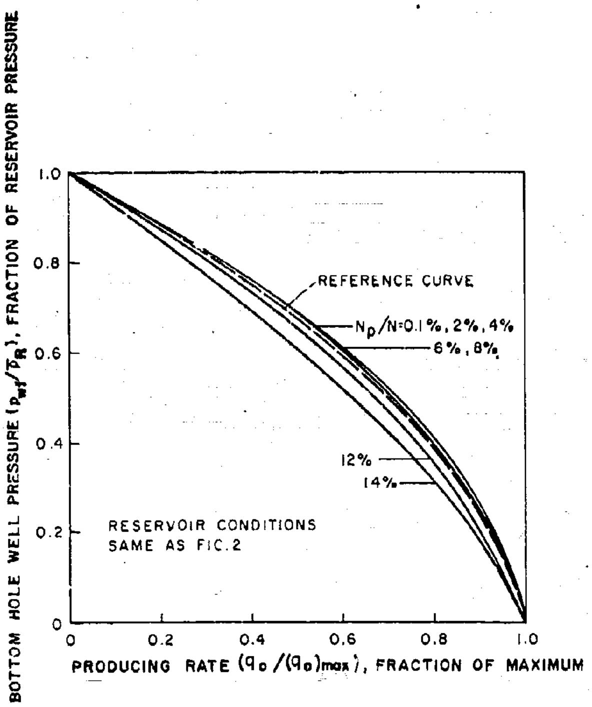

Fig. 6—*Comparison of reference curve with computer-calculated IPR curves.*

Effect of Relative Permeability and Other Conditions

The same basic shape of the curves was noted when the study was extended to cover a much wider range of conditions. Runs were made with three different sets of relative permeability curves in various combinations with the different crude oils. The results were in agreement sufficient to indicate that the relationship might be valid for most conditions.

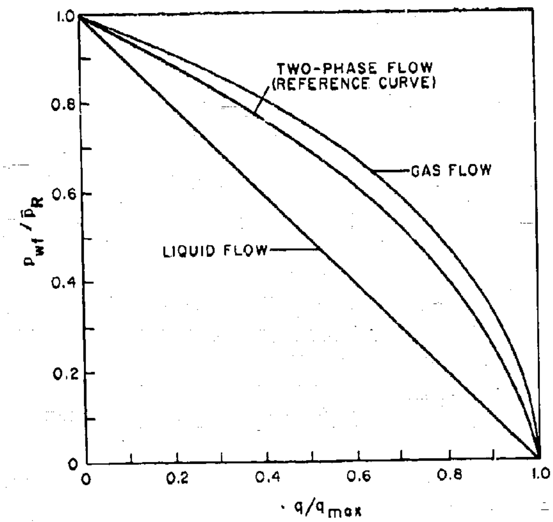

Fig. 8—*Comparison of IPR's for liquid flow, gas flow and two-phase flow.*

To explore further the generality of the relationship, a run was made in which the crude oil PVT curves and the relative permeability curves were roughly approximated by straight lines. It was surprising to find that, even with no curvature in either the graphs of crude oil characteristics or the relative permeability input data, the output IPR's exhibited about the same curvature as those from previous computer runs.

Calculations also were made for different well spacings, for fractured wells and for wells with positive skins. Good agreement was noted in all cases except for the well with a skin effect, in which case the IPR's more nearly approached straight lines.

In summary, calculations for 21 reservoir conditions resulted in IPR's generally exhibiting a similar shape.

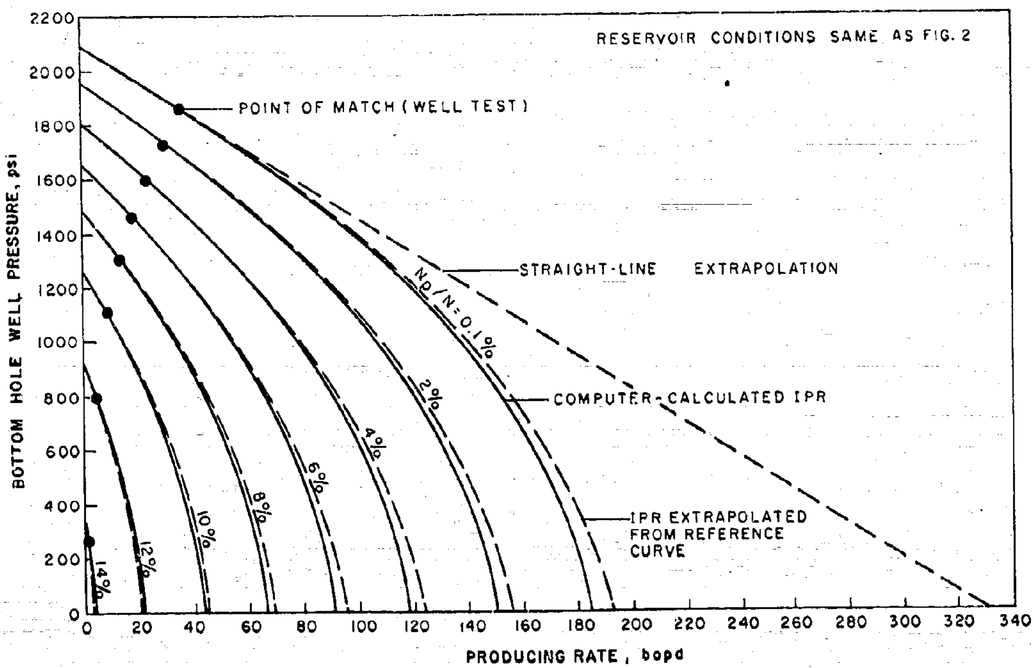

Fig. 7—*Deviations when IPR's are predicted by reference curve from well tests at low drawdowns.*

Significant deviation was noted only for the more viscous crude, for a reservoir initially above the bubble point, and for a well producing through a restrictive skin. Even in these cases, definite curvature was still apparent.

The curves of crude oil characteristics and of relative permeability that furnished the input data for the various conditions studied are given in Appendix A. Dimensionless IPR curves calculated for various conditions are shown in Appendix B.

Proposed Reference IPR Curve

If the IPR curves for other solution-gas drive reservoirs exhibit the same shape as those investigated in this study, well productivities can be calculated more accurately with a simple reference curve than with the straight-line PI approximation method currently used.

Applying one reference curve to all solution-gas drive reservoirs would not imply that all these reservoirs are

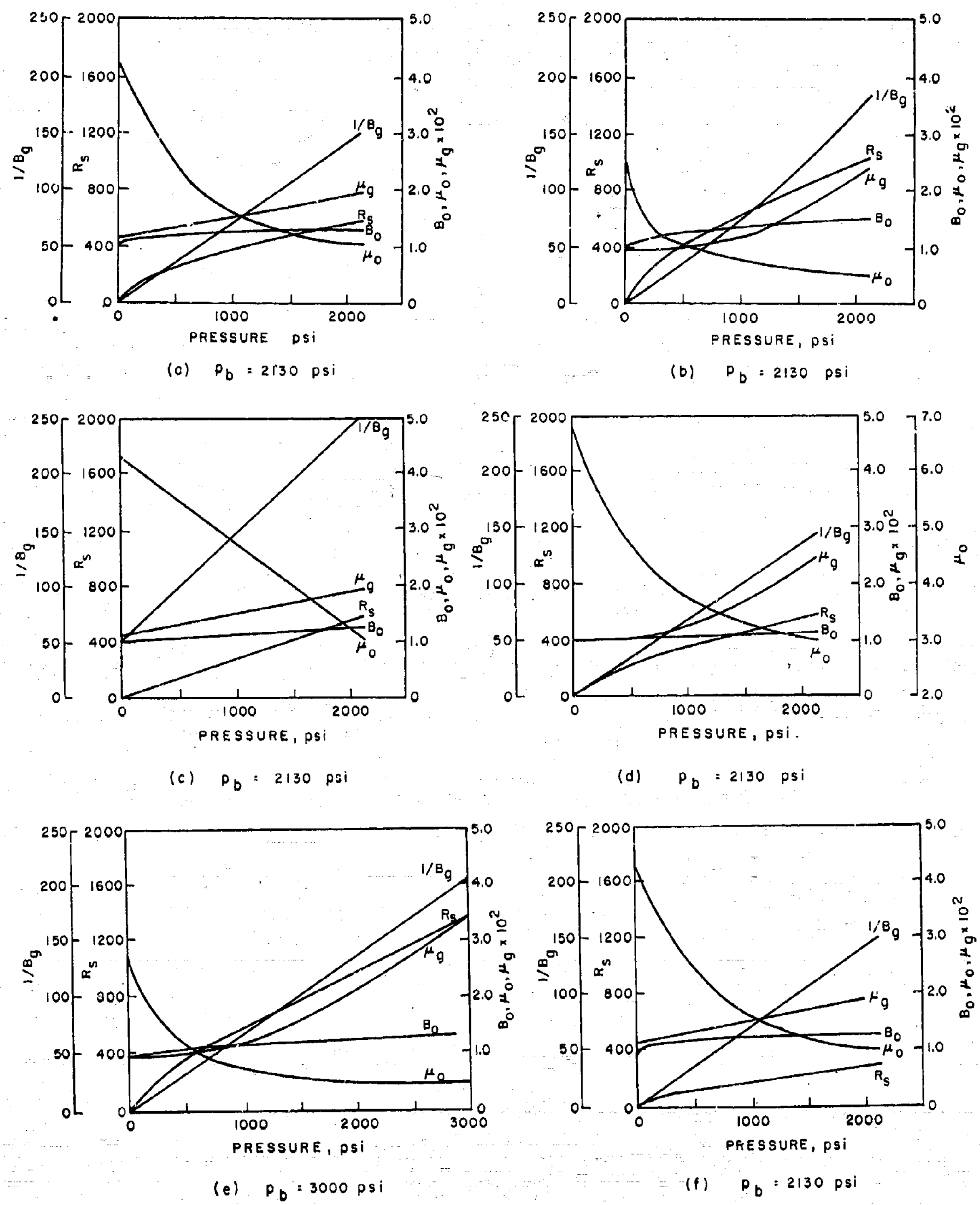

Fig. 9—*Input data, crude oil PVT characteristics ($c_o = 1.2 \times 10^{-5}$ in all cases).*

identical any more than would the present use of straight-line PI's for all such reservoirs. Rather, the curve can be regarded as a general solution of the solution-gas drive reservoir flow equations with the constants for particular solutions depending on the individual reservoir characteristics.

Although one of the dimensionless curves taken from the computer calculations could probably be used as a reference standard, it seems desirable to have a mathematical statement for the curve to insure reproducibility, permanency and flexibility in operation.

The equation of a curve that gives a reasonable empirical fit is

$$\frac{q_o}{(q_o)_{max}} = 1 - 0.20\frac{p_{wf}}{\bar{p}_R} - 0.80\left(\frac{p_{wf}}{\bar{p}_R}\right)^2 \quad . \quad . \quad . \quad . \quad (1)$$

where q_o is the producing rate corresponding to a given well intake pressure p_{wf}, $\bar{p}_R$ is the corresponding reservoir pressure, and $(q_o)_{max}$ is the maximum (100 percent drawdown) producing rate. Fig. 5 is a graph of this curve.

For comparison, the relationship for a straight-line IPR is

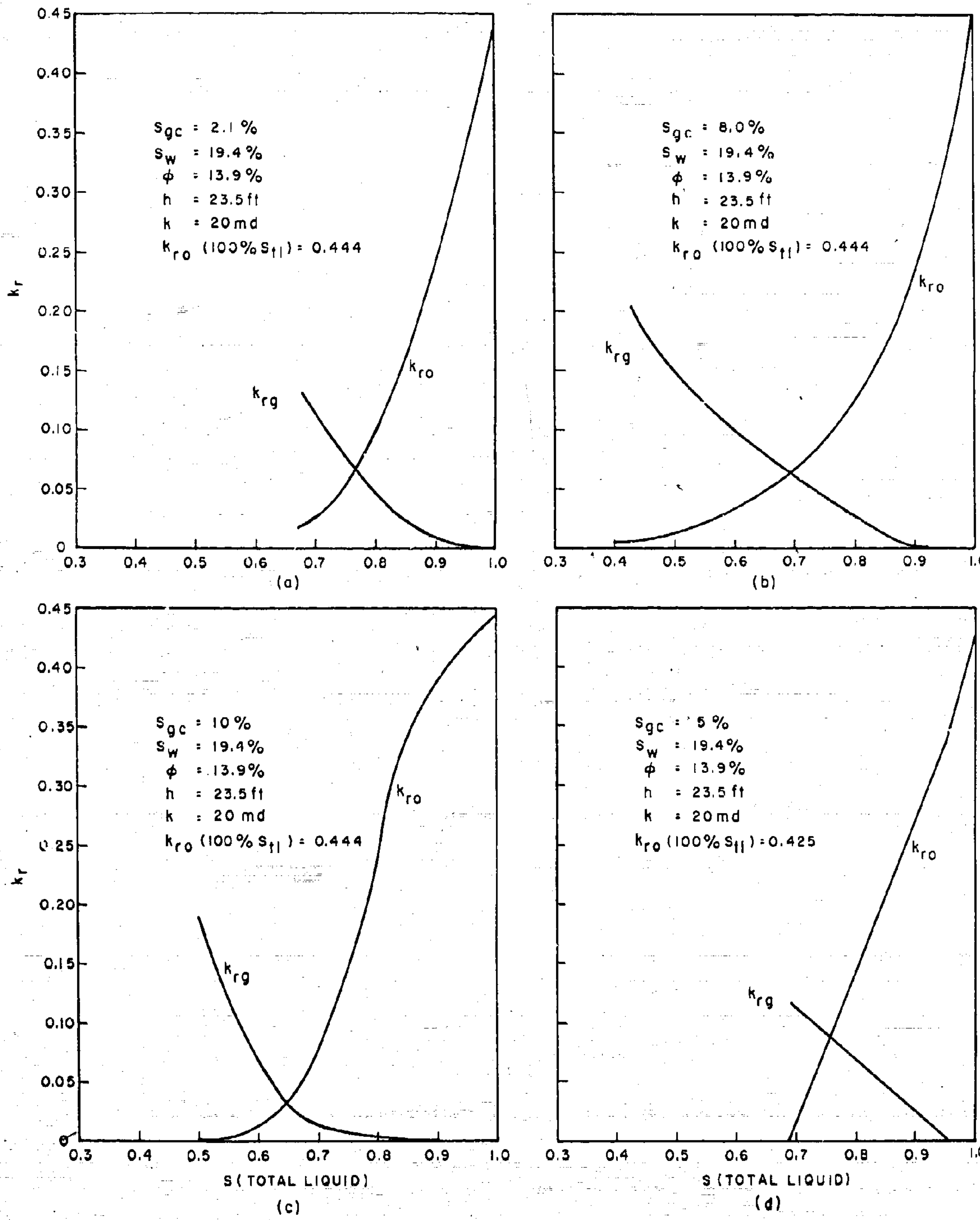

Fig. 10—*Input data, relative permeability curves.*

$$\frac{q_o}{(q_o)_{max}} = 1 - \frac{p_{wf}}{\bar{p}_R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

When $q_o/(q_o)_{max}$ from Eq. 1 is plotted vs $p_{wf}/\bar{p}_R$, the dimensionless IPR reference curve results. On the basis of the cases studied, it is assumed that about the same curve will result for all wells. If q_o is plotted vs p_{wf}, the actual IPR curve for a particular well should result.

A comparison of this curve with those calculated on the computer is illustrated in Fig. 6. The curve matches more closely the IPR curves for early stages of depletion than the IPR curves for later stages of depletion. In this way, the percent of error is least when dealing with the higher producing rates in the early stages of depletion. The percentage error becomes greater in the later stages of depletion, but here production rates are low and, as a consequence, numerical errors would be less in absolute magnitude.

Use of Reference Curve

The method of using the curve in Fig. 5 is best illustrated by the following example problem. A well tests 65 BOPD with a flowing bottom-hole pressure of 1,500 psi in a field where the average reservoir pressure is 2,000 psi. Find (1) the maximum producing rate with 100 percent drawdown, and (2) the producing rate if artificial lift were installed to reduce the producing bottom-hole pressure to 500 psi.

The solution is: (1) with $p_{wf} = 1{,}500$ psi, $p_{wf}/\bar{p}_R = 1{,}500/2{,}000 = 0.75$. From Fig. 5, when $p_{wf}/\bar{p}_R = 0.75$, $q_o/(q_o)_{max} = 0.40$, $65/(q_o)_{max} = 0.40$, $(q_o)_{max} = 162$ BOPD; (2) with $p_{wf} = 500$ psi, $p_{wf}/\bar{p}_R = 500/2{,}000 = 0.25$. From Fig. 5, $q_o/(q_o)_{max} = 0.90$, $q_o/162 = 0.90$, $q_o = 146$ BOPD.

If the same calculations had been made by straight-line PI extrapolation, the productivity with artificial lift would have been estimated as 195 BOPD rather than 145 BOPD. This illustrates a significiant conclusion to be drawn for cases in which such IPR curvature exists. Production increases resulting from pulling a well harder will be less than those calculated by the straight-line PI extrapolation; conversely, production losses resulting from higher back pressures will be less than those anticipated by straight-line methods.

It is difficult to overstate the importance of using stabilized well tests in the calculations. In a low-permeability

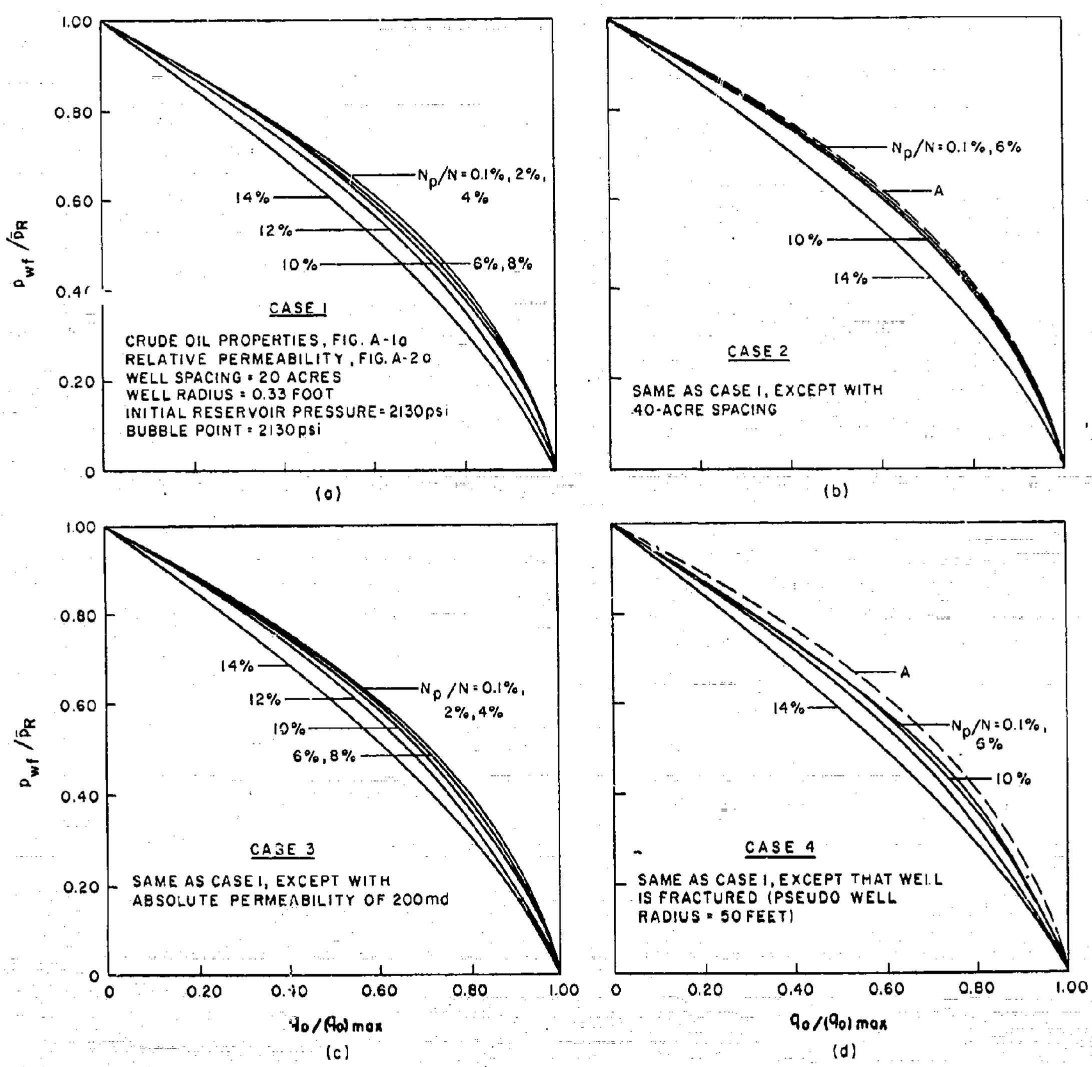

Fig. 11—*Calculated dimensionless IPR curves.*

reservoir it frequently will be found that significant changes in producing conditions should not be made for several days preceding an important test. This presents no problem if a well is to be tested at its normal producing rate, but it becomes more difficult if multi-rate tests are required.

Accuracy of Reference Curve

It is anticipated that the most common use of the reference IPR curve will be to predict producing rates at higher drawdowns from data measured at lower drawdowns. For example, from well tests taken under flowing conditions, predictions will be made of productivities to be expected upon installation of artificial lift. It is necessary to arrive at the approximate accuracy of such predictions.

Maximum error will occur when well tests made at very low producing rates and correspondingly low drawdowns are extrapolated with the aid of the reference curve to estimate maximum productivities as the drawdown approaches 100 percent of the reservoir pressure. The error that would result under such conditions was investigated, and typical results are shown in Fig. 7. In this figure the dashed lines represent IPR's estimated from well tests at low drawdowns (11 to 13 percent), and the solid lines represent the actual IPR's calculated by the computer. The maximum error for the reservoir considered in Fig. 7 is less than 5 percent throughout most of its producing life, rising to 20 percent during final stages of depletion. Although the 20 percent error may seem high, the actual magnitude of the error is less than ½ BOPD.

It is obvious from Fig. 7 that if well tests are made at higher drawdowns than the extreme cases illustrated, the point of match of the estimated and actual IPR curves is shifted further out along the curves and better agreement will result.

Maximum-error calculations were made for all the reservoir conditions investigated. Except for those cases with viscous crudes and with flow restricted by skin effect, it appears that a maximum error on the order of 20 percent should be expected if all solution-gas drive IPR's follow the reference curve as closely as have the several cases investigated. For comparison, the maximum errors for the straight-line PI extrapolation method were generally between 70 and 80 percent, dropping to about 30 percent only during final stages of depletion.

The figures cited above refer to the maximum errors that should be expected. In most applications the errors should be much less (on the order of 10 percent) be-

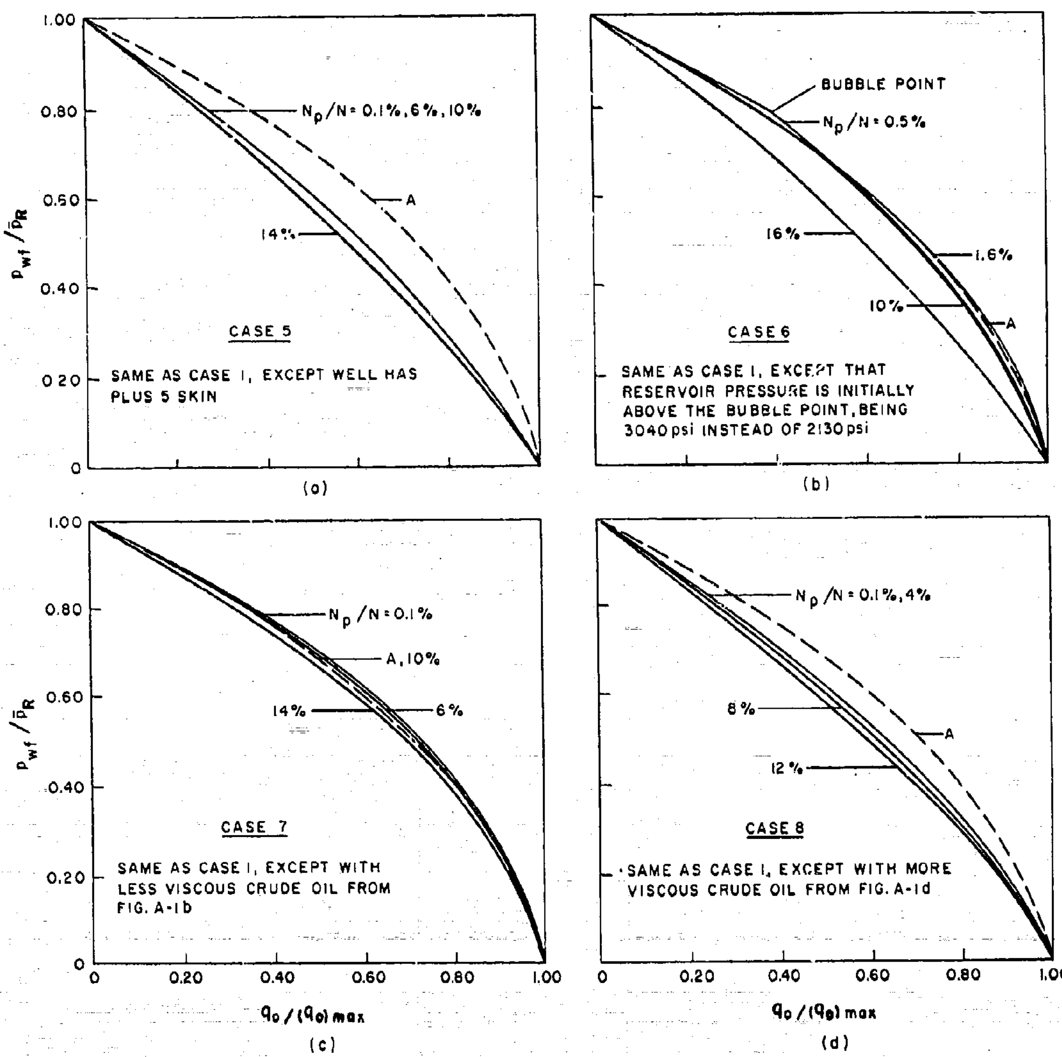

Fig. 12—*Calculated dimensionless IPR curves.*

cause better agreement is noted between IPR's and reference curve throughout most of the producing life of the reservoirs and because well tests are ordinarily made at greater drawdowns.

Application of Reference Curve: Other Types of Reservoirs

The proposed dimensionless IPR curve results from computer analysis of the two-phase flow and depletion equations for a solution-gas drive reservoir only and would not be considered correct where other types of drive exist. In a major field with partial water drive, however, there can be large portions of the field that are effectively isolated from the encroaching water by barrier rows of producing wells nearer the encroachment front. It appears that the reference curve could be used for the shielded wells for at least a portion of their producing lives. Similarly, the reference curve might give reasonable results for a portion of the wells producing from a reservoir in which expansion of a gas cap is a significant factor.

Since the reference curve is for the two-phase flow of oil and gas only, it would not be considered valid when three phases (oil, gas and water) are flowing. However, it appears intuitively that some curvature should be expected in the IPR's whenever free gas is flowing in a reservoir. For radial flow, this curve should lie somewhere between the straight line for a single-phase liquid flow and the curve for single-phase gas flow. The dimensionless IPR's for the two types of single-phase flow are compared with the suggested reference curve for solution-gas drive reservoirs in Fig. 8.

Conclusions

IPR curves calculated both for different reservoirs and for the same reservoirs at different stages of depletion varied several-fold in actual magnitude. Nevertheless, the curves generally exhibited about the same shape.

This similarity should permit substitution of a simple empirical curve for the straight-line PI approximations commonly used. Maximum errors in calculated productivities are expected to be on the order of 20 percent compared with 80 percent with the PI method. Productivity calculations made with the reference curve method rather than with the PI method will show smaller production increases for given increases in drawdowns and, conversely, less lost production for given increases in backpressures.

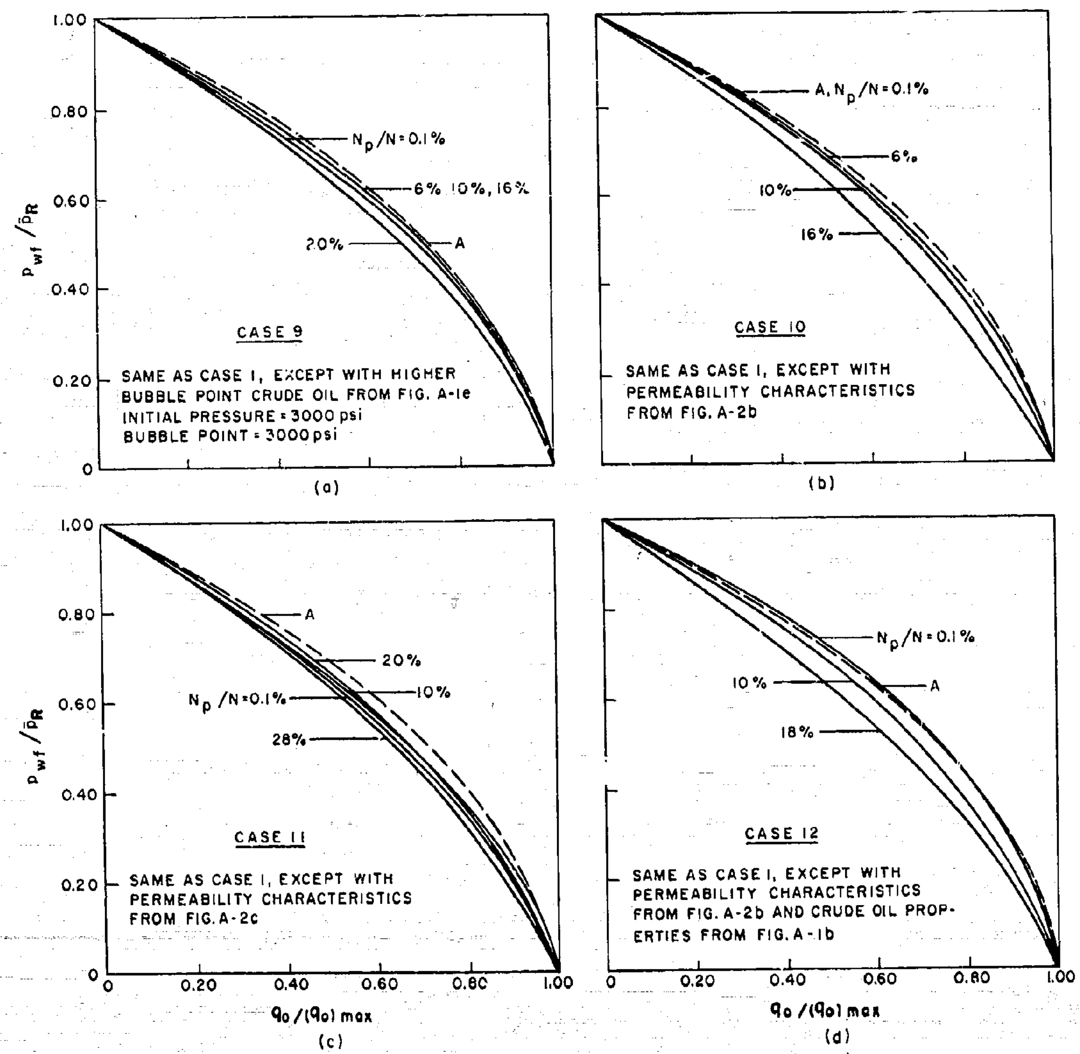

Fig. 13—*Calculated dimensionless IPR curves.*

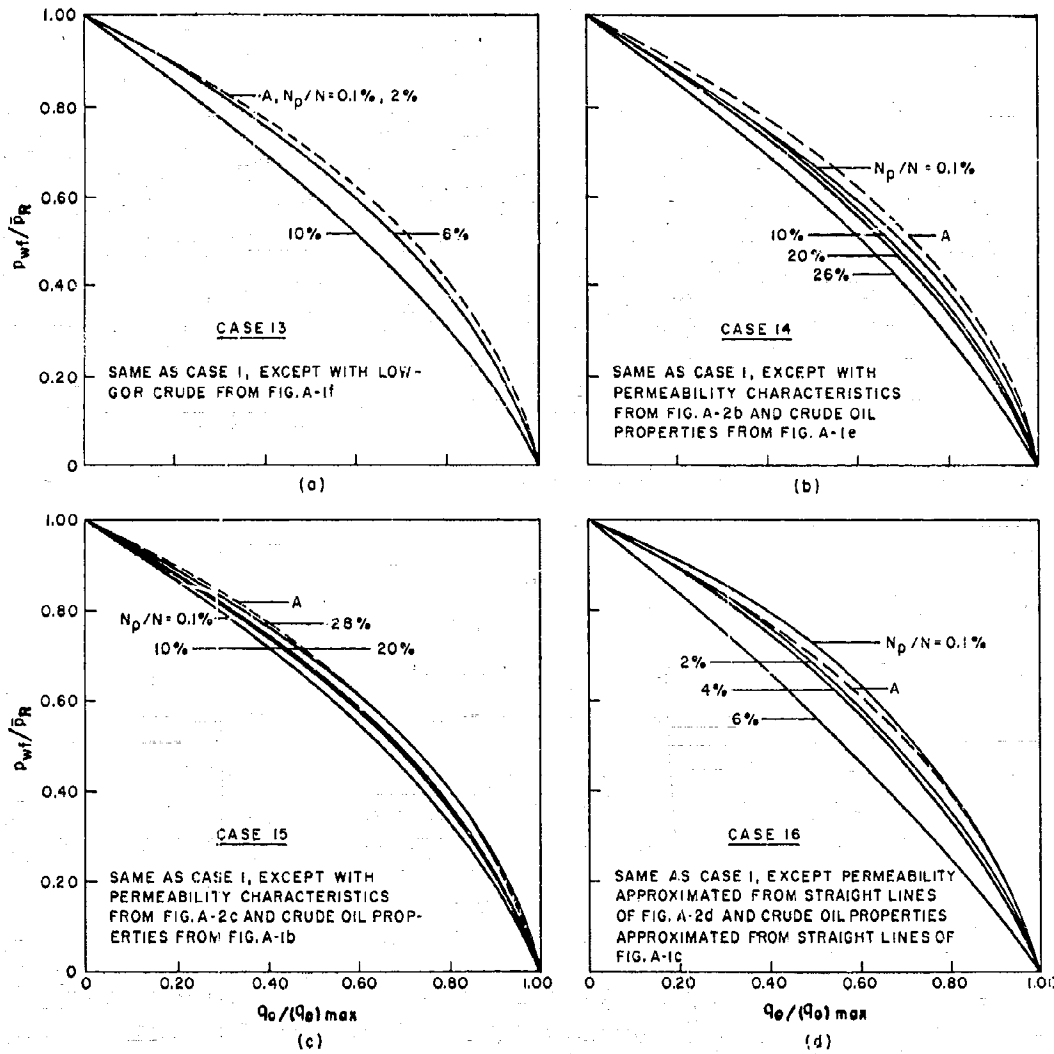

Fig. 14—*Calculated dimensionless IPR curves.*

This technique needs to be verified by a comparison with field results. As previously discussed, the conclusions are based only on computer solutions involving several simplifying assumptions as listed in the Introduction.

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APPENDIX A

Input Data

Figs. 9 and 10 illustrate graphically the input data (crude oil PVT characteristics and relative permeability characteristics) from which the theoretical behavior of simulated reservoirs was calculated by the computer.

APPENDIX B

Computer-Calculated IPR Curves

Dimensionless IPR Curves

Figs. 11 through 14 are graphs of the theoretical IPR's calculated for various simulated reservoir conditions. So that the IPR's under various conditions can be compared more easily, the initial IPR curve ($N_p/N = 0.1$ percent) from Fig. 11a is reproduced on all succeeding figures and is designated as Curve A.

In addition to the cases illustrated, five more calculations were made in which individual curves of the crude oil properties in Fig. 9a were replaced one by one with the curves from Fig. 9b. The results were comparable to those shown, and, since the illustrations include the case in which the curves of Fig. 9a were completely replaced by those of Fig. 9b, it was not considered necessary to reproduce the cases in which the individual components were replaced. ★★★

Editor's note: A picture and biographical sketch of J. V. Vogel appear on page 60.