Preschool Math Tutorial

Introduction

One of the most important tasks of a Directional Drilling Engineer is to NAVIGATE the borehole into a specific target. The common navigation techniques involve measuring :

Distance
Direction
Inclination

These measurements are then used to calculate the position of the wellbore in terms of coordinates (Rectangular or Polar) and Vertical Depth .

We can reduce the incidence of mistakes in Navigational Calculations by using computers to perform them. Computers are faster than people , they are accurate and do not get upset or tired. They do however occasionally break down.

A Directional Driller, without a rigsite computer, must be able to continue navigating using more primitive methods of computation. The DD's tool is Mathematics, or more specifically, Geometry and Trigonometry. These are the essential basics of the DD's trade and all Directional Drillers must be familiar with them. Well planning, Survey calculations and Trajectory Analysis all require a basic understanding of Trigonometry and Geometry.

While the level of Geometry and Trig needed to perform DD calculations is basic and fairly simple, many Directional Drillers may never have learned them , or more commonly, may have forgotten much of the Geometry and Trig that they learned in High School or College. The purpose of this Pre School Tutorial is to review the basic principles as either a refresher course for those who have forgotten or an introduction to those who have never done them before. Candidates coming to Delta DD1 and 3 schools must have the basic skills in order to learn some of the Navigational Techniques which will be taught.

The tutorial begins with very basic concepts. If you feel that you do not need to review any particular section, I suggest that you at least skim through it to refresh your memory. I also suggest that you complete the exercises on your own before referring to the attached answer sheet.

1) Formulae in ALGEBRA

Equations

In Algebra, an equation or formula is nothing more than a statement.

For example:
In order to mix the color Green, an Artist must mix the colors Blue and Yellow. He can express this as a Formula :

Green = Blue + Yellow

To mix a Darker Green his formula may have to changed to:

Dark Green = 2Blue + 1Yellow

Or two times the amount of Blue than Yellow

The formulæ used in Arithmetic and Algebra are concerned more with numbers than colors, however they still express true statements.

For example:

$$5 = 3 + 2$$

or

$$27 = 10 \times 2 + (3+4)$$

Equations in ALGEBRA substitute letters for unknown numerical values. For example:

Instead of writing $5 = 3 + 2$

I write $5 = 3 + X$

and ask " What is the value of X ?" - Obviously X equals 2

Or I could substitute A for 5 and write:

$$A = 3 + X$$

In this case the value of A will depend on the value of X
So I could then ask " What is the value of A, if X equals 6 ? "
Again the answer should obviously be (3 + 6) or 9
Here is another illustration :

$$R = \frac{180}{3b} \qquad \text{What is R, if } b = 10 \text{ ?}$$

answer: $$R = \frac{180}{30} = 6$$

One important thing to remember about an equation is that it must be balanced. That is the total value of the expressions on one side of the equal sign must be the same as the total values on the other side. For example:

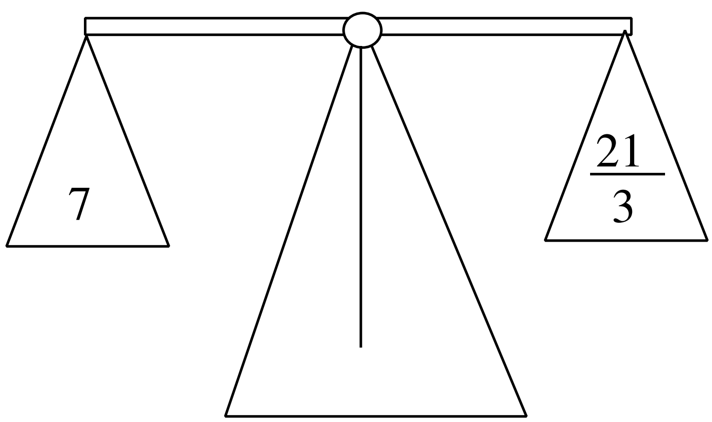

Fig 1.1

$$7 = 21 \div 3$$

This is an important comparison, because one can do anything to one side of an equation providing that the same thing is done to the other side. If the scales in fig 1.1 above were to have 7 on one side and 21 ÷ 4 on the other, the equation would be unbalanced and untrue :

$$7 \text{ ? } 21 \div 4$$

Supposing however that we were to do the same thing to both sides of the equation ..... multiply them both by 2 for example. We would then have the following:

$$7 \times 2 = \frac{21 \times 2}{3}$$

This equation is balanced and true.

The same applies to equations where only letters or symbols are used. The following examples are true for the same values : $A = B^2$

$$2A = 2B^2$$

$$2A + 27 = 2B^2 + 27$$

$$\frac{2A + 27}{4276} = \frac{2B^2 + 27}{4276}$$

Because we are doing the same thing to both sides of the equation, they are all balanced and true. (Providing of course that the original equation " $A = B^2$ " is true)

Another technique used in equations is the ability to "Cancel Out".

This can best be shown by the following example:

$\frac{A}{B}$ This means $A \div B$

If I multiply this expression by B, I get the following :

$$\frac{A}{B} \times B$$

In Mathematics, if an expression is multiplied and divided by the same number, these actions cancel each other out and the net effect is nothing (the same applies to adding and subtracting the same number - net effect - nothing).

So the expression $\frac{A}{B} \times B$ really equals just plain A

Because the "B"s have cancelled each other out.
This, coupled with the ability to modify both sides of an equation, allows us to rearrange equations into formats which are more suitable.

e.g. If I have the equation $$A + 32 = \frac{B}{4}$$

I can adjust this in several ways. I can move the value 32 to the right hand side of the equation if I change it to -32; giving :

$$A = \frac{B}{4} - 32$$

Suppose I want to express B in terms of A (ie I want B alone on one side of the equation), I can move the Expression ÷ 4 to the left side of the equation and change its sign from ÷ to x. So:

$$(A + 32) \times 4 = B$$

or correctly :

$$B = (A + 32) \times 4$$

Notice that the expression (A + 32) has been enclosed in brackets. This means that the value of A + 32 must first be added before being multiplied by 4.

Without the brackets, the equation would read :

$$B = A + 32 \times 4$$

or

$$B = A + 128$$

In Mathematics, operations must take place in a certain, logical order. Expressions in brackets must be calculated first. Multiplication or division is next and finally addition or subtraction. This is very important as not adhering to this rule can result in several different answers to the same problem, as can be seen in the example above.

Exercises

1) For the values :

$A = 6.57$
$B = 2.43$

Solve the following equations :

$$X = A^2 + 7.32$$

$$X = A + \frac{A}{B}$$

$$X = \sqrt{A + B}$$

$$X = A^2 + B^2$$

$$X = \sqrt{A^2 + B^2}$$

$$X = 3A + 4B$$

2) If : $A = 3B$

What does $\frac{4A}{3}$ equal in terms of B ?

3) In the following equation :

$$R = \frac{180}{r \times p}$$

Solve R, where $r = 0.05$,

4) In the following equation :

$$X = \frac{Y \times Z}{0.5}$$

Solve Y where : $X = 18.8$, $Z = 2$

5) What is the value of the expression :

$$A^2 + \frac{A}{B} \times 2$$

Where $A = 4$, $B = 3.721$

6) Solve :

$$A = 16 + 42 \div 6$$

$$A = (469 - 318) \times (42 + 24)$$

$$A + 94 = 360 \div 2$$

$$A = 75 \times \frac{64}{25}$$

2) Basics of GEOMETRY

Geometry is the study of the relationships between lines and circles. These will be considered by themselves and in relation to each other.

Lines and Angles

When one straight lines meets another, they form two angles (fig 2.1)

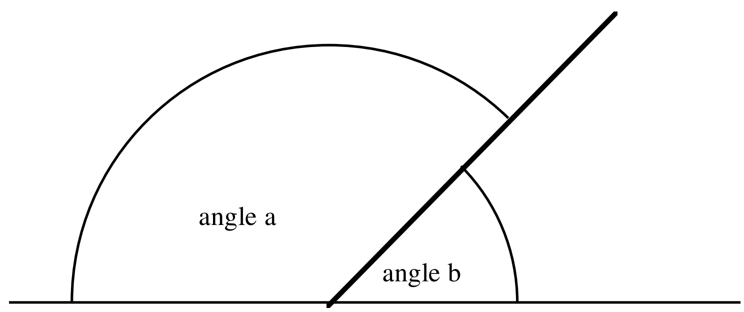

Fig 2.1

On a straight line, the sum of all the angles about a point is always 180°. In the above example (fig 2.1), the sum of the two angles a & b is 180°. Therefor we can say :

angle a = 180 - angle b

and

angle b = 180 - angle a

These are known as *supplementary angles.*

Where two straight lines cross or *intersect* (fig 2.2), the sum of angles around the point of intersection is always 360°.

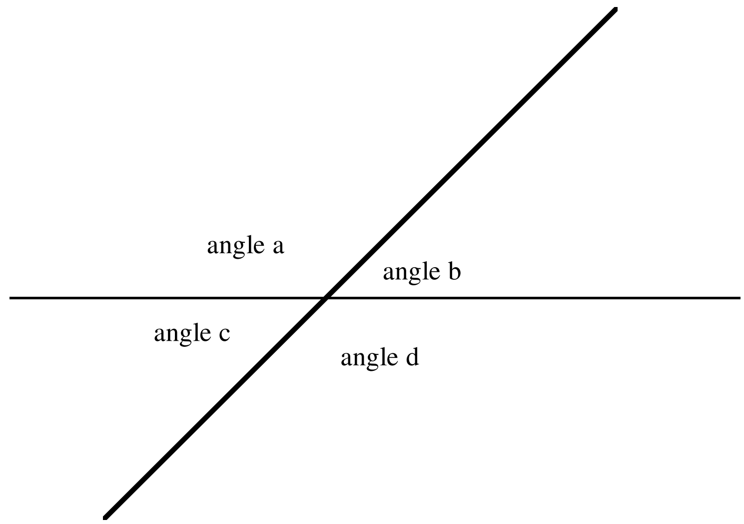

Fig 2.2

The *opposite* angles are equal.

i.e. a = d and b = c

By knowling the value of one angle, we can easily calculate the others.

e.g.

If angle a = 135°
angle b = 180 - 135 Therefor angle b = 45°
angle c = angle b Therefor angle c = 45°
angle d = angle a Therefor angle d = 135°

Lines which always remain the same distance from each other and never meet are said to be *parallel* (fig 2.3)

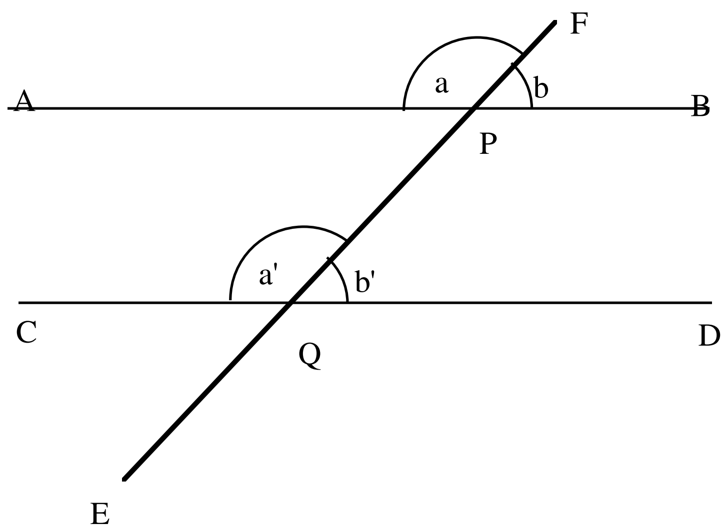

Fig 2.3

By naming the ends of lines and intersection points with letters, we are able to describe the lines and angles more clearly. For example in fig 2.3 :

Line $\overline{AB}$ is parallel to line $\overline{CD}$

Lines $\overline{AB}$ and $\overline{CD}$ are intersected by line $\overline{EF}$ at points P and Q

The angle a can be referred to as angle FPA
The angle b can be referred to as angle FPB

Note that the straight line $\overline{EF}$ intersects the two parallel lines, $\overline{AB}$ and $\overline{CD}$ at the same angle such that :

$$\text{angle } a = a'$$

$$\text{angle } b = b'$$

Polygons

Polygons are multi-sided figures. Each side of a polygon is a straight line. Fig 2.4 shows examples of some commonly found polygons.

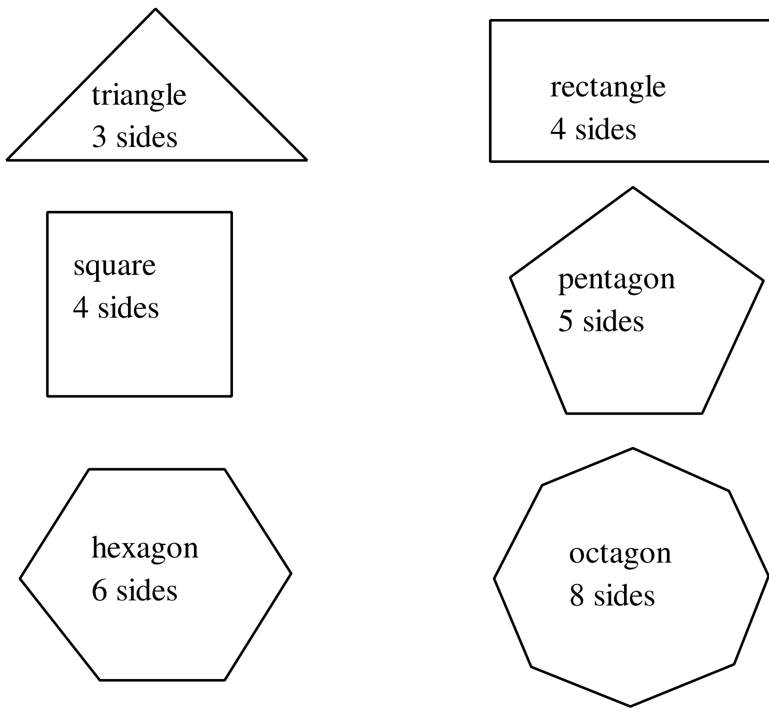

fig 2.4

For all polygons, we can state that the sum of the *interior angles* is equal to :

$$180 \times (n-2)$$

where n = the number of sides

eg

The sum of the interior angles of an hexagon = 180 x (6-2) = 720°
The sum of the interior angles of a triangle = 180 x (3-2) = 180°

Triangles

The simplest polygon, and the one which most concerns us is the Triangle. All polygons can be divided into triangles, so by understanding triangles, we can solve most problems concerning other polygons.

Triangles can assume an infinite number of shapes, depending on the lengths of the three sides. (fig 2.5)

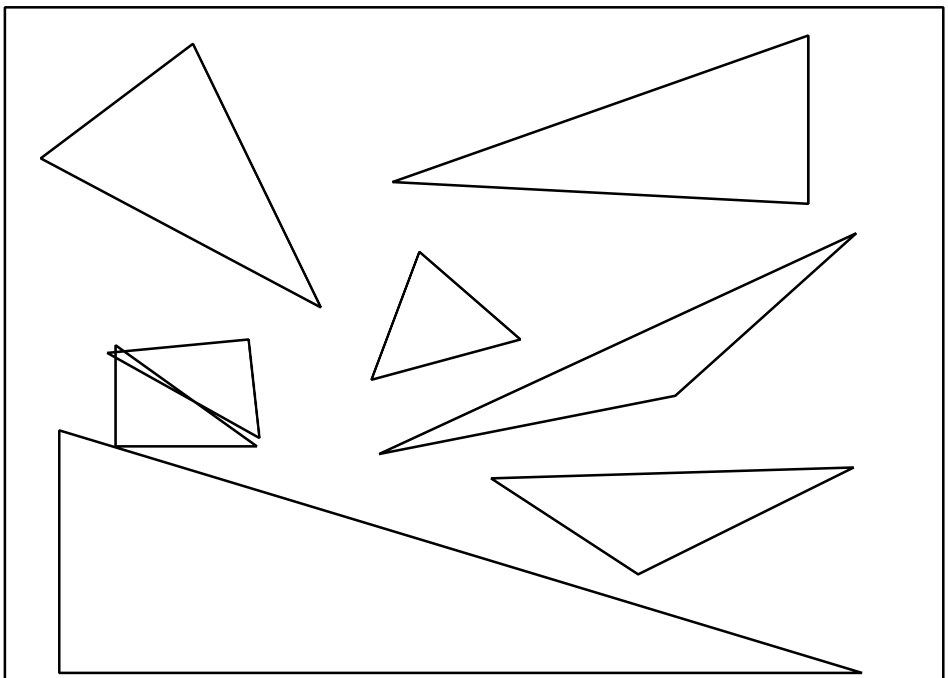

fig 2.5

Regardless of the size or shape of the triangle, the sum of its interior angles will always be 180°

Knowing this, we can calculate the size of any angle if we know the size of the other two (fig 2.6)

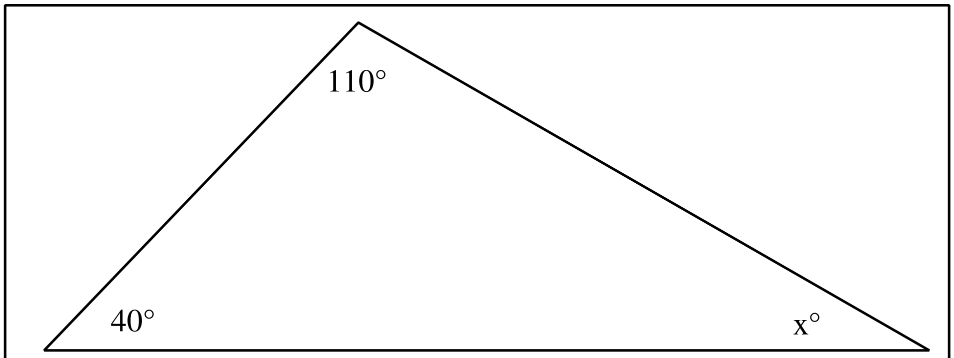

Fig 2.6

$$110° + 40° + x° = 180°$$
$$x = 180 - (110 + 40)$$
$$x = 30°$$

There is a type of triangle which interests us more than others. This is the *Right Angled Triangle.*

A right angled triangle is any triangle having one angle equal to 90° (fig 2.7). Consequently, he sum of the remaining two (complementary) angles is also 90°. As can be seen in fig 2.7, these can be of various shapes and sizes.

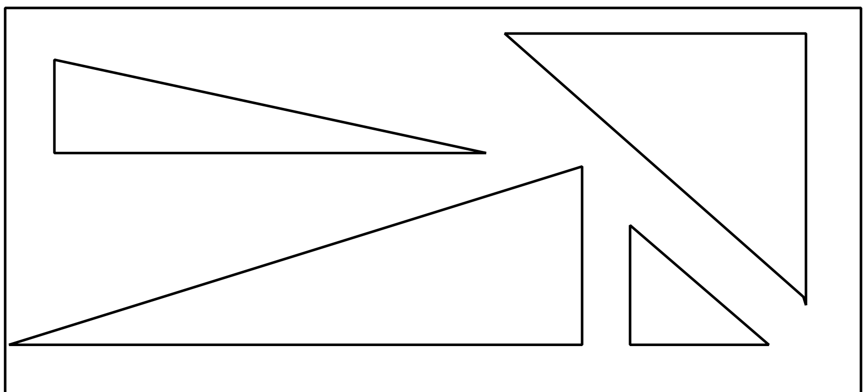
fig 2.7

Circles

The circle is the most commonly encountered regular geometric shape. A regular circle (perfectly round) is very interesting from a mathematical point of view.

First, and most obvious, circles are always the same shape regardless of their size.

We can measure circles in one of three ways :

Circumference the distance around the outside of the circle

Diameter the distance across the circle

Radius the distance from the center of the circle to the circumference

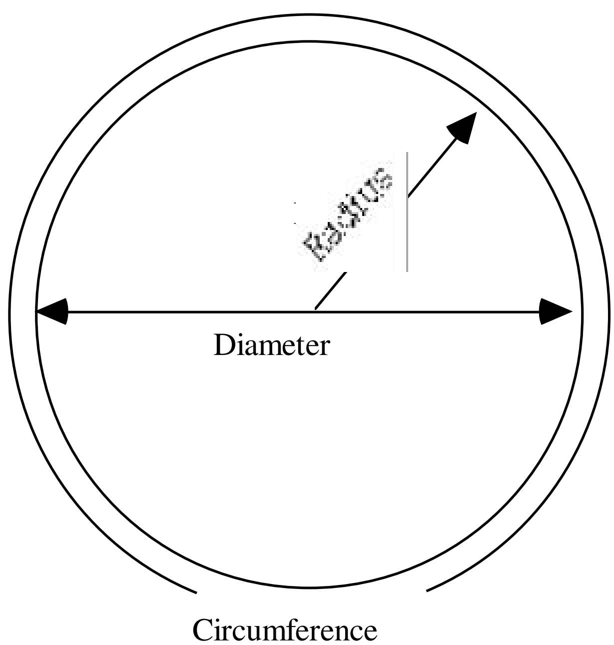

fig 2.8

Any one of these measurements can be used to define the size of the circle.

The diameter is always equal to twice the length of the radius :

$$D = 2R$$

$$R = \frac{D}{2}$$

In a perfect circle, the Circumference divided by the Diameter always gives the same number, regardless of the size of the circle. This number can be calculated out to an infinite number of decimal places, so for ease of use it is normally rounded to four decimal places.

Because it always has the same value it is referred to as a *Constant*

The Ancient Greeks, who discovered this relationship, referred to this constant as *Pi* (p), pronounced "pie" in English, which is the Greek letter "P".

The rounde value of p is 3.1416 *

*(most hand held calculators have this value installed as a separate keystroke)

Knowing the value of this constant becomes extremely useful when we investigate circles or part of a circle.

For example we know that :

$$p = \frac{C}{D}$$

therefor $$C = D \times p$$

Knowing this we can calculate the circumference of a circle having a diameter of 6 inches :

$$C = 6 \times p$$
$$C = 6 \times 3.1416$$

$$C = 18.85 \text{ inches}$$

Knowing that the Diameter of a circle is twice the radius. we can substitute D with 2R to give :

$$C = 2R \text{ x } p$$

or as is more commonly expressed :

$$C = 2pR$$

<u>Arcs</u>

Circles can be divide into segments or parts known as *Arcs.*

An Arc is normally expressed in terms of the radius of the circle from which it was derived, and the angular change it proscribes.
(fig 2.9)

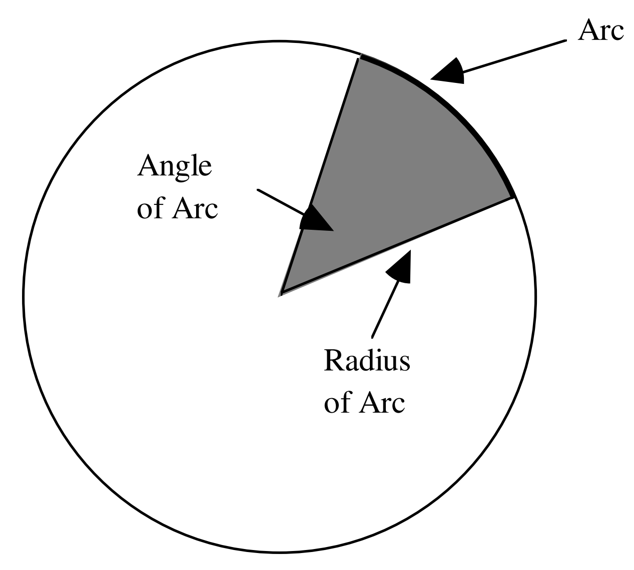

fig 2.9

For any given Radius, the length of the arc is determined by the angle of arc.

<u>Tangents</u>

A Tangent is any line which touches the Circumference of a circle at one point only.

Tangents are always perpendicular (ie at 90°) to the radius of the circle they are touching.

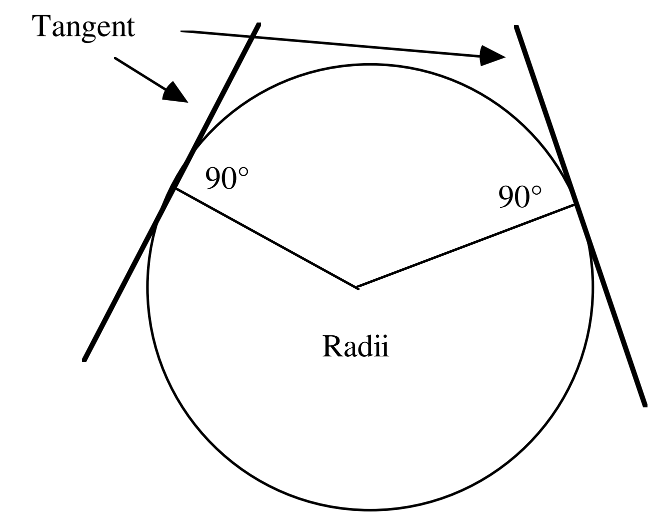

fig 2.10

Chapter 2 Exercises

1) In figure 1 , if angle AQF = 115°, solve :

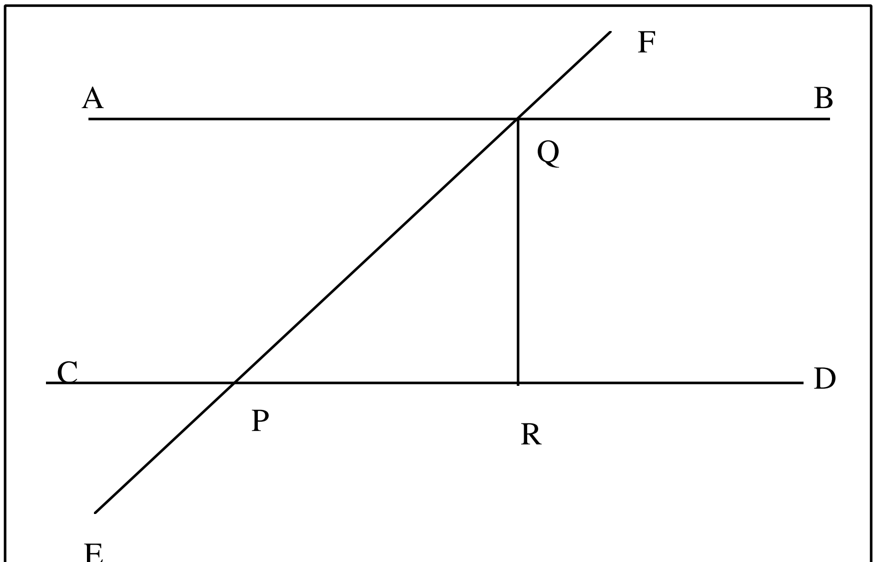

fig 1

Angles	FQB	=	
	AQP	=	
	EPR	=	
	EPC	=	
	PQR	=	

2) In figure 2, solve :

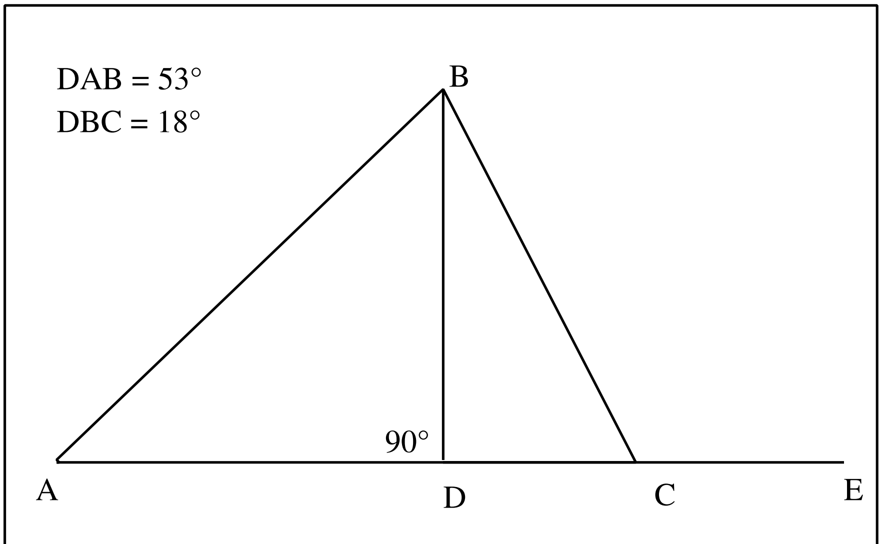

fig 2

Angles	ABD	=	
	DCB	=	
	BCE	=	

3) If a bicycle has 26" diameter wheels, how far will it travel for each complete revolution of the wheel ?

4) If the Circumference divided by the diameter of a 10 inch diameter circle equals 3.1416, what value for the same calculation would a 5 inch diameter circle give ?

5) What is the sum of the internal angles of a pentagon (5 sided polygon) ?

3) Basics of TRIGONOMETRY

Trigonometry is based on the relationships of the sides and angles of Right Angled triangles.
The shape of any triangle is determined by the angles within it, while the size of the triangle is determined by the lengths of the respective sides. Any two triangles having the same internal angles will have the same shape regardless og their size. These triangles are called *Similar triangles.* (fig 3.1)

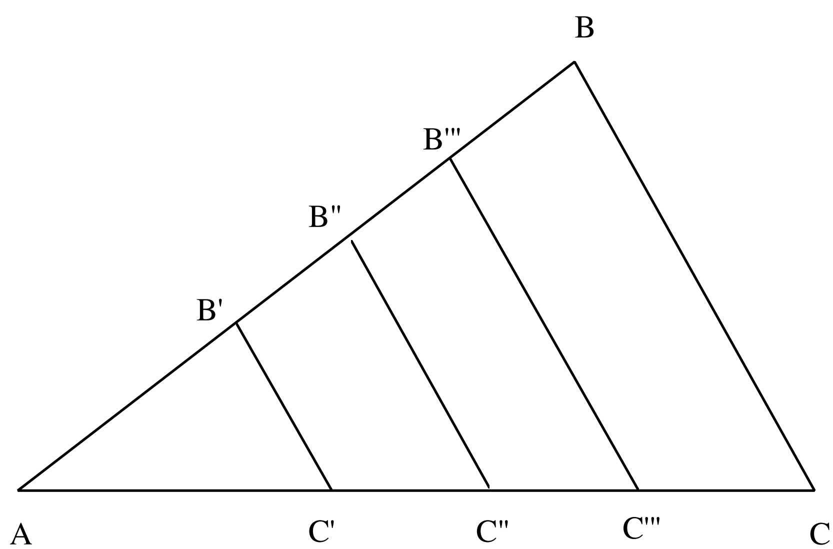

fig 3.1

The ratio of the sides of similar triangles remains constant :

$$\frac{AB}{AB'} = \frac{BC}{B'C'} = \frac{AC}{AC'}$$

and

$$\frac{AB}{AC} = \frac{AB'}{AC'} = \frac{AB''}{AC''}$$

etc...

Pythagorean Theorem

Pythagoras, an ancient Greek mathematician, discovered that for any right angles triangle the length of the side opposite the right angle (which he called the *Hypotenuse*) squared is equal to the sum of the squares of the other two sides.

eg

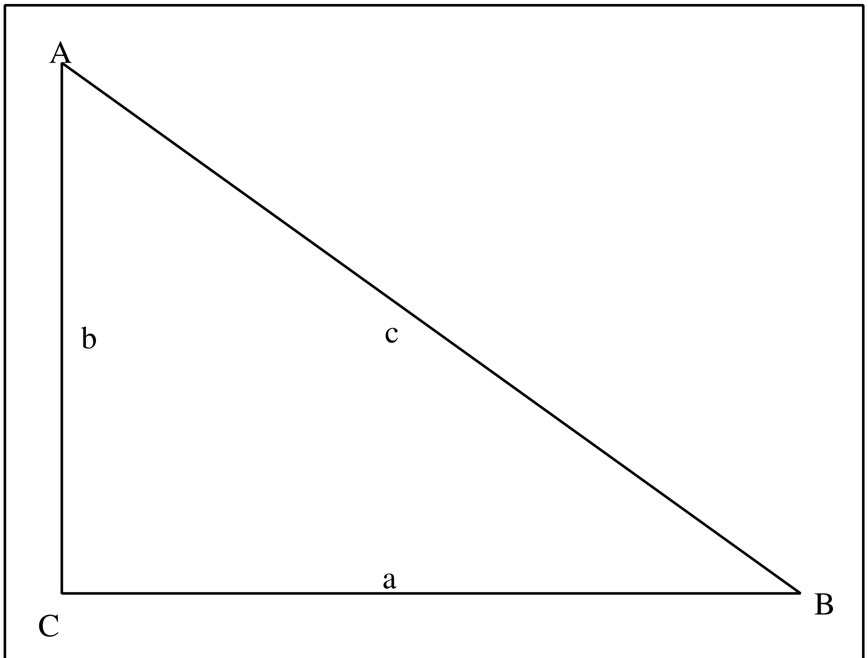

fig 3.2

$$c^2 = a^2 + b^2$$

Thus knowing the lengths of any two sides of a right angle triangle, we can calculate the lenght of the third side.

For example in fig 3.2 :

if side a = 4 and side b = 3

then side $c^2 = 4^2 + 3^2$

Or $c = \sqrt{16 + 9}$

Side c = 5

The ancient Greeks also discovered that right angled triangles having the same complementary angles, have constant ratios between their sides and that the angles themselves are a function of the length of the sidesa. Each of the complementary angles is defined by the Hypotenuse side and an Adjacent side. The side opposite to the angle is called the Opossite side. The opposite side for one of the complementary angles is the Adjacent side for the other.

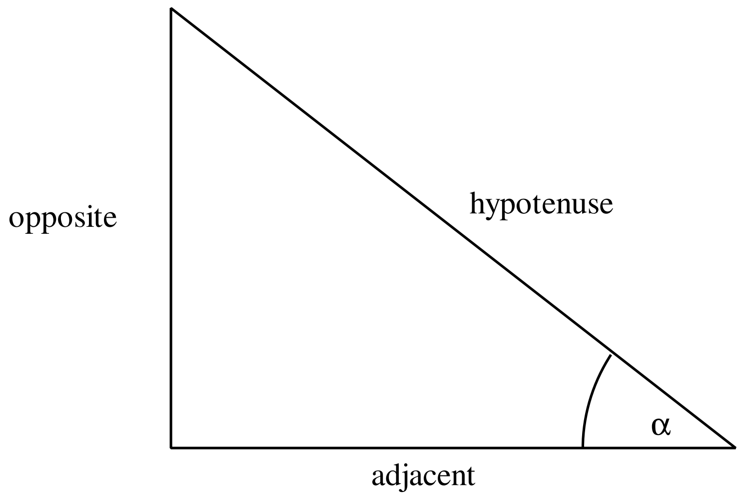

fig 3.3

These relationships form the basis of trigonometry.

They are expressed as a set of primary Ratios

Cosine α = $\dfrac{\textit{Adjacent}}{\textit{Hypotenuse}}$

Sine α = $\dfrac{\textit{Opposite}}{\textit{Hypotenuse}}$

Tangent α = $\dfrac{\textit{Opposite}}{\textit{Adjacent}}$

For example if the Hypotenuse of the triangle is known, say 150 feet and the angle α is 37° :

the adjacent side	= Cos 37 x 150
	= 119.8 feet
the opposite side	= Sin 37 x 150
	= 90.27 feet

the remaining angle must equal 90 - 37 which is 53°

So, for a right angle triangle, given any 2 sides or angles, or one of each, we can use trigonometric ratios to calculate the remaining angles or sides.

Chapter 3 exercises

1)

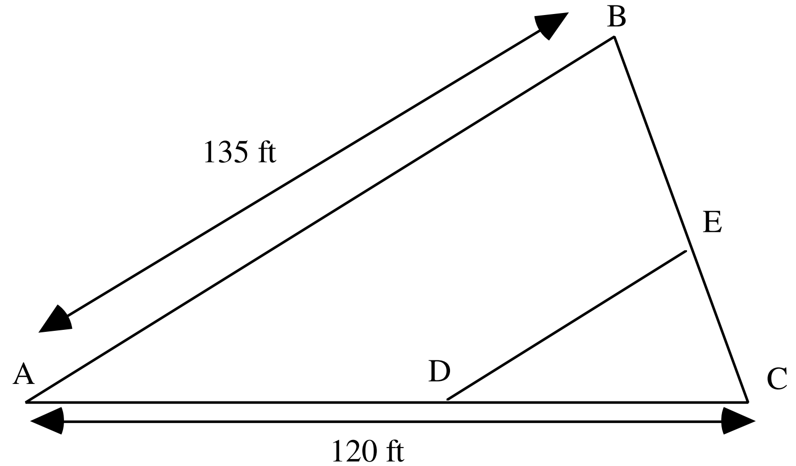

fig 1

In fig 1, if line DC is 40 feet long, how long is line DE ?

2)

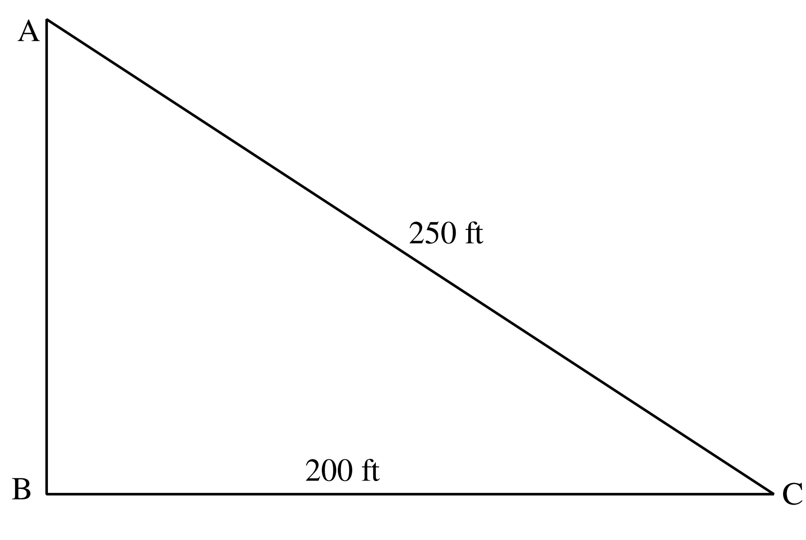

fig 2

What is the length of line AB ?

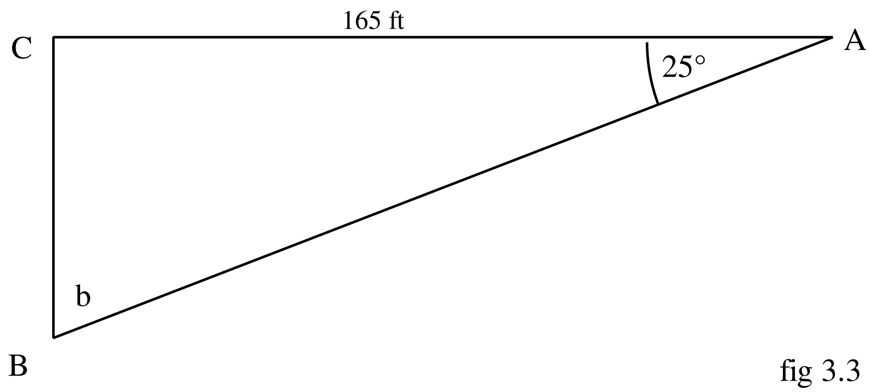

fig 3.3

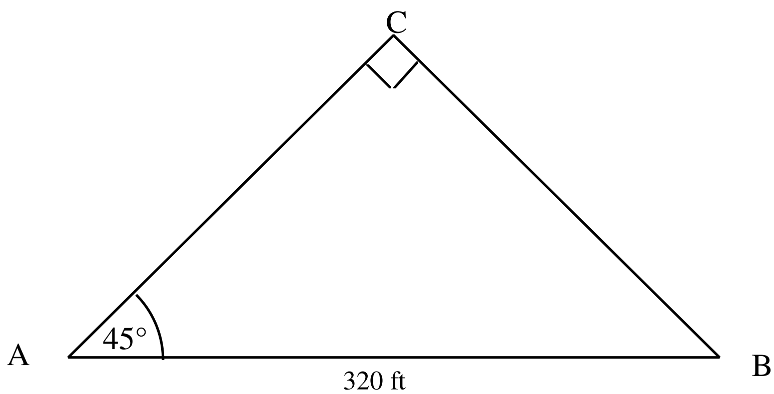

fig 3.4

Solve figs 3.3 and 3.4 for the remaining unknown angles and sides.

Answers to Exercises

Chapter 1

1)	a)	50.48	2)	4B	3) 1145.9
	b)	9.27			
	c)	3			
	d)	49.07			
	e)	7			
	f)	29.43			

4) 4.7

5) 18.15

6) a) 23
b) 9966
c) 86
d) 192

Chapter 2

1)	a)	FQB = 65°	2)	ABD = 37°	3) 81.68 in
	b)	AQP = 65°		DCB = 72°	
	c)	EPR = 115°		BCE = 108°	
	d)	EPC = 65°			
	e)	PQR = 25°			

4) 3.1416

5) 540

Chapter 3

1) 45 ft long

2) 150 ft

3) a) BA = 182.06 ft
BC = 76.94 ft
angle b = 65°

b) AC = 226 27
BC = 226.27
angle CBA = 45°