Factors Affecting the Angle of Inclination and Dog-Legging in Rotary Bore Holes†

ARTHUR LUBINSKI* AND H. B. WOODS[♁]

ABSTRACT

When conditions are not varied during the drilling of a bore hole, the hole assumes a constant angle of inclination. When conditions vary, the hole changes direction and eventually assumes another angle. Mathematical relationships are established giving the forces on the bit, the dog-legging tendency, and the inclination toward which the hole tends as functions of drill-collar size, hole size, weight on bit, dip of the formation, stabilizers, etc. Conclusions are reached concerning economical means of fighting crooked-hole conditions.

INTRODUCTION

One of the co-authors presented a paper on buckling of rotary drilling strings[1] at the 1950 spring meeting of the Mid-Continent District. Later, the other co-author pointed out that the previous study concerned a straight and vertical hole and suggested that a similar study be made for inclined holes not only in homogeneous formations, but also in stratified formations, either horizontal or inclined. This was done in the present paper.

In the first part of the paper, the forces acting on a bit are analyzed. Later, the properties of the formations being drilled will be studied; and they, combined with the forces acting on the bit, will be used to determine the relation between the angle or change of angle of inclination of the bore hole and the factors that influence them.

This paper is based on mathematical derivations which are given in the appendix.

DIRECTION OF THE FORCE ON BIT

Consider a drilling string in a straight, but not vertical, hole as shown in Fig. 1. At the vicinity of the bit, the string does not contact the wall of the hole. At some distance above the bit, the drilling string contacts the wall of the hole at a point designated as "point of tangency." Above the point of tangency, the string lies on the low side of the hole.

* Stanolind Oil and Gas Co., Tulsa.

♁ Hughes Tool Co., Houston.

† Presented by Arthur Lubinski at the spring meeting of the Mid-Continent District, Division of Production, Tulsa, March 1953.

[1] References are at the end of the paper.

With no weight on the bit, the only force acting on the bit is the result of the weight of the portion of the string between the bit and the tangency point. This force tends to bring the hole toward vertical. When weight is applied, there is another force on the bit which tends to direct the hole away from vertical. The resultant of these two forces may be in such a direction as to increase angle, to decrease angle, or to maintain constant angle.

The derivation of a relationship between the direction of the force on the bit and all the variables that affect it is given in the appendix. The relationship itself is plotted in Fig. 2.

This derivation is based upon the assumption that the drilling string lies on the low side of the hole as shown in Fig. 1. Actually, for some conditions, the drilling string will not lie along the low side, but will buckle helically. It will be shown later herein that in actual drilling conditions this situation occurs only exceptionally and the foregoing assumption is correct.

In this study, dynamic forces caused by rotation are disregarded. The justification for this is the fact that, as may be seen later, the conclusions are confirmed by actual field facts.

Consider first the example case of Fig. 2, which refers to a drilling string with $6\frac{1}{4}$-in. collars in $8\frac{3}{4}$-in. hole. The upper abscissa is the angle of inclination α of the hole with respect to the vertical as shown in Fig. 1. The ordinate is the ratio of the angle ϕ of the inclination of the force on bit to the angle α of the inclination of the hole (see Fig. 1). The curves correspond to various values of the weight on the bit. Consider, for instance, that 18,000 lb are carried in a hole inclined 10 deg. Point D of curve 2 indicates that for these conditions ϕ/α is equal to 0.93, which means that ϕ is smaller than α; and, consequently, the force on the bit tends to decrease the hole inclination.

For drill collars other than $6\frac{1}{4}$-in., and for hole diameters other than $8\frac{3}{4}$-in., the lower abscissa scale of Fig. 2 must be used. This abscissa represents the value of $\alpha\, m/r$, in which:

- α is the hole inclination in radians (1 radian = 57.3 deg).

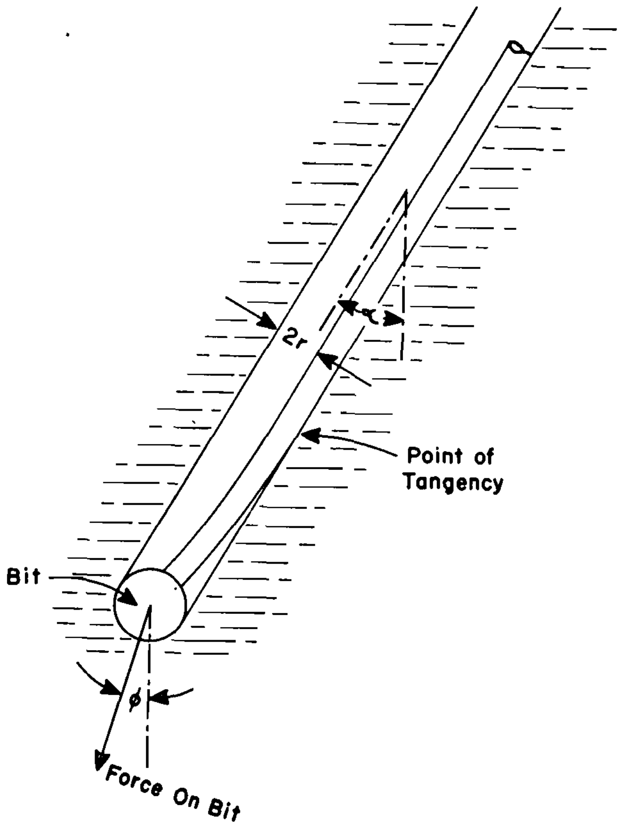

Fig. 1

r is the apparent radius of the hole in feet (one-half of diametral clearance between hole and collars).

m is the length in feet of the one dimensionless unit. Values of m are obtainable from Fig. 3.

m depends upon the size of drill collars and buoyancy. Fig. 3 was prepared from a formula given in a previous publication.[1] The inside diameter was assumed equal to 2¼ in. for 6-in. OD drill collars, and proportional to outside diameter for other sizes. The influence of inside diameter on values of m is, in any event, not very significant.

When using the lower abscissa scale, then various curves correspond to various weights on the bit, not in pounds, but in dimensionless units of weight. The weight of one dimensionless unit is given in Fig. 4 for various sizes of drill collars and mud densities.

The location of the point of tangency is shown in Fig. 5 in which the meaning of the two abscissa scales (general case and example case) are the same as in Fig. 2 and the curves correspond to the same values of the weight on the bit. The left and right ordinate scales represent the distance between the bit and the tangency point in dimensionless units (general case) and in feet (example case), respectively.

HOLE INCLINATION IN ISOTROPIC FORMATIONS

We shall assume that in a formation with no bedding planes, i.e. in an isotropic* formation, a bit drills in the direction of the force which is imparted to it.

Consequently, the direction of the force on the bit given in Fig. 2 becomes the direction in which the hole is drilled.

Consider, for instance, the condition represented by the point A of Fig. 2. This point lies on curve 1, for which the weight on the bit is equal to 36,000 lb. The upper abscissa scale shows that the hole inclination is equal to 10 deg. The corresponding value of ϕ/α, as seen in the ordinate scale, is less than unity. Consequently, when drilling ahead, the inclination of the hole will decrease. If the weight on the bit of 36,000 lb is maintained, then the drilling conditions will be represented in succession by various points of curve 1 located to the left of A until the condition represented by point B is reached, for which $\phi/\alpha = 1$, which means that the hole inclination will not change any more. This inclination is equal to 2.25 deg.

Consider now the case of the same weight on the bit as previously, but in a hole close to vertical, such as represented by point C, for which the inclination is equal to 0.25 deg. This time, ϕ/α is greater than unity; and, while drilling ahead, the angle of inclination will build up and successive conditions will be represented by points of curve 1 located to the right of C until again the condition represented by B is reached, for which the inclination is equal to 2.25 deg. Thus, it may be seen that with 6¼-in. drill collars in an 8¾-in. hole and with 36,000 lb on the bit, the hole inclination will approach 2.25 deg irrespective of its initial inclination. In other words, point B corresponds to a stable drilling condition. The angle toward which the hole tends for a given set of conditions will be referred to as "equilibrium angle."

If, on the other hand, 18,000 lb were carried instead of 36,000 lb, then points D, E, and F of curve 2 must be considered in place of points A, B, and C of curve 1. Point D is located farther from the line $\phi/\alpha = 1$ than point A, which means that in a hole inclined 10 deg, the tendency to bring the hole toward vertical is stronger for 18,000 lb than for 36,000 lb. Similarly, point F is located closer to the line $\phi/\alpha = 1$ than point C, which means that in a hole inclined 0.25 deg, the tendency to build up angle is smaller when 18,000 lb are carried than with 36,000 lb.

* "Isotropic" means having properties identical in all directions.

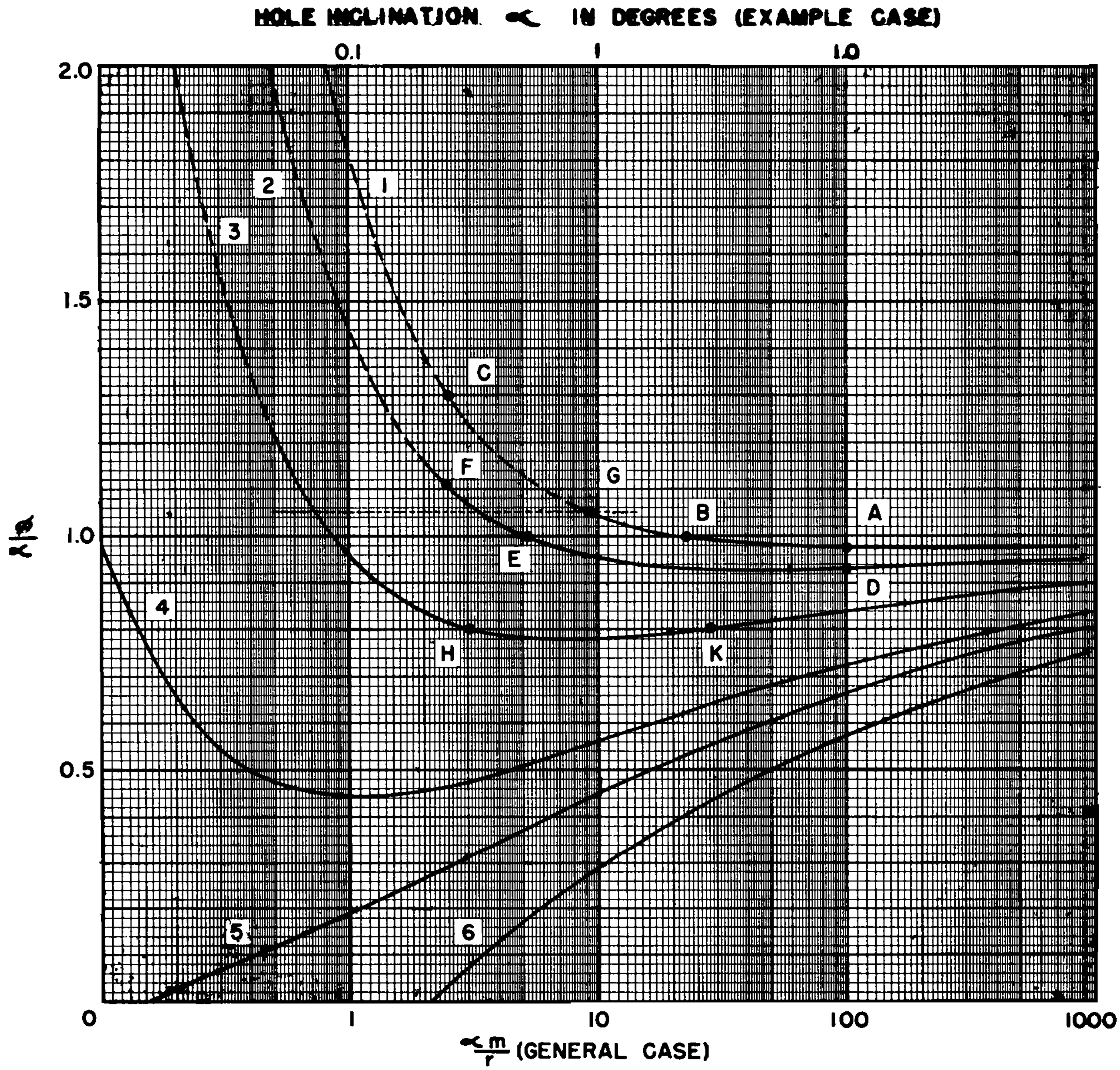

Fig. 2–Ratio of Angle ϕ of Inclination of the Force on Bit over Angle α of Hole Inclination

	Curve 1	2	3	4	5	6
Example Case:						
6¼-in. drill collars, 8¾-in. hole, 10 lb per gal mud.						
Weight in pounds	36,000	18,000	9,000	5,400	4,500	3,600
General Case:						
Weight in dimensionless units	8	4	2	1.2	1	0.8

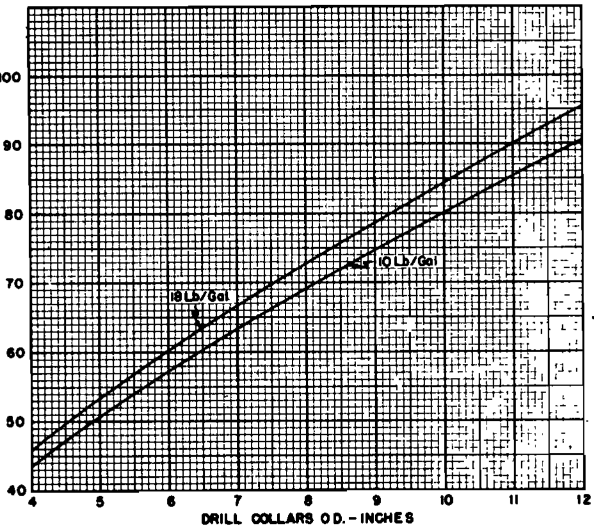

Fig. 3–Length *m* of One Dimensionless Unit

If 18,000 lb are carried instead of 36,000 lb, then curve 2 shows that the angle of inclination toward which the hole tends is indicated by point E and is equal to 0.5 deg.

Curves 3 and 4 indicate that, for the values of the weight equal respectively to 9,000 and 5,400 lb, the inclination of the hole should tend to 0.09 and 0.01 deg, respectively; which, for all practical purposes, means a vertical hole. However, it is interesting to note that from a theoretical standpoint for still less weight, e.g., 4,500 lb as shown by curve 5, the hole does not tend to any very small value of inclination, but comes to vertical.

In Fig. 6 values of $\propto m/r$ for equilibrium conditions taken from Fig. 2 were plotted (curves $h = 0$*) vs. the weight on the bit in dimensionless units. For a given size of drill collar, m is a constant and $\propto m/r$ is proportional to the equilibrium angle per unit of apparent radius of the hole (half of the clearance).

Substituting numerical values for various sizes of drill collars and drill pipe in place of dimensionless values of Fig. 6, curves of Fig. 7 were plotted. Here the equilibrium angle in degrees per inch of clearance between collars (or tool joints) and hole was plotted vs. the weight on the bit in pounds for 6¼-in. drill collars, 8⅛-in. drill collars, 10½-in. drill collars, and 4½-in. drill pipe.

Consider, for instance, the case of 6¼-in. drill collars and 45,000 lb on the bit. Point A of Fig. 7 indicates that this inclination is equal to about 1½ deg per inch of clearance between collars and hole, which corresponds to an inclination of $1.5 \times 2.75 = 4\frac{1}{8}$ deg in a 9-in. hole. On the other hand, point B of Fig. 7 indicates—for instance—that for 8⅛-in. drill collars, the hole tends toward an inclination of 0.3 deg per inch of clearance, which corresponds to only 0.26 deg in a 9-in. hole, as compared to 4⅛ deg for 6¼-in. collars in the same hole. These examples show that holes drilled with large drill collars are much closer to vertical than holes drilled with the same weight and conventional collars.

Point C shows that carrying only 15,000 lb with drill pipe and no collars at all results in an inclination greater than 7 deg per inch of clearance or more than 20 deg in a 9-in. hole, which explains the necessity for drill collars.

Although problems concerning hole inclination in inclined beds are much more important from a practical standpoint, it is interesting to point out that the common belief that holes go inclined only because of inclined formations or lack of symmetry of the drilling string is erroneous. Even in isotropic formation, a perfectly vertical hole cannot be drilled with an elastic drilling string, unless extremely small and uneconomical weights are used.

DRILLING ANISOTROPY INDEX

Consider the case of a straight, but inclined, hole represented in Fig. 8 and a set of conditions for which the force on the bit makes an angle with the vertical. It is well known that inclined bedding planes generally make the bit drill updip in hard rocks such as encountered in West Texas, Mid-Continent, and Rocky Mountains. This is shown in Fig. 8. Consequently, the direction of drilling is different from the direction of the force on the bit. For such to be the case, the formation must have a slightly lower drillability along the bedding planes than perpendicular to them. The relative difference

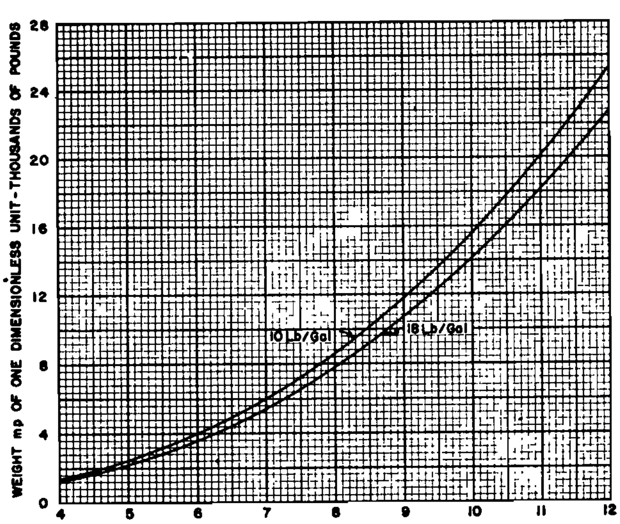

Fig. 4–Weight of One Dimensionless Unit

* The definition of h will be given further in this paper, $h = 0$ means that the formation is isotropic.

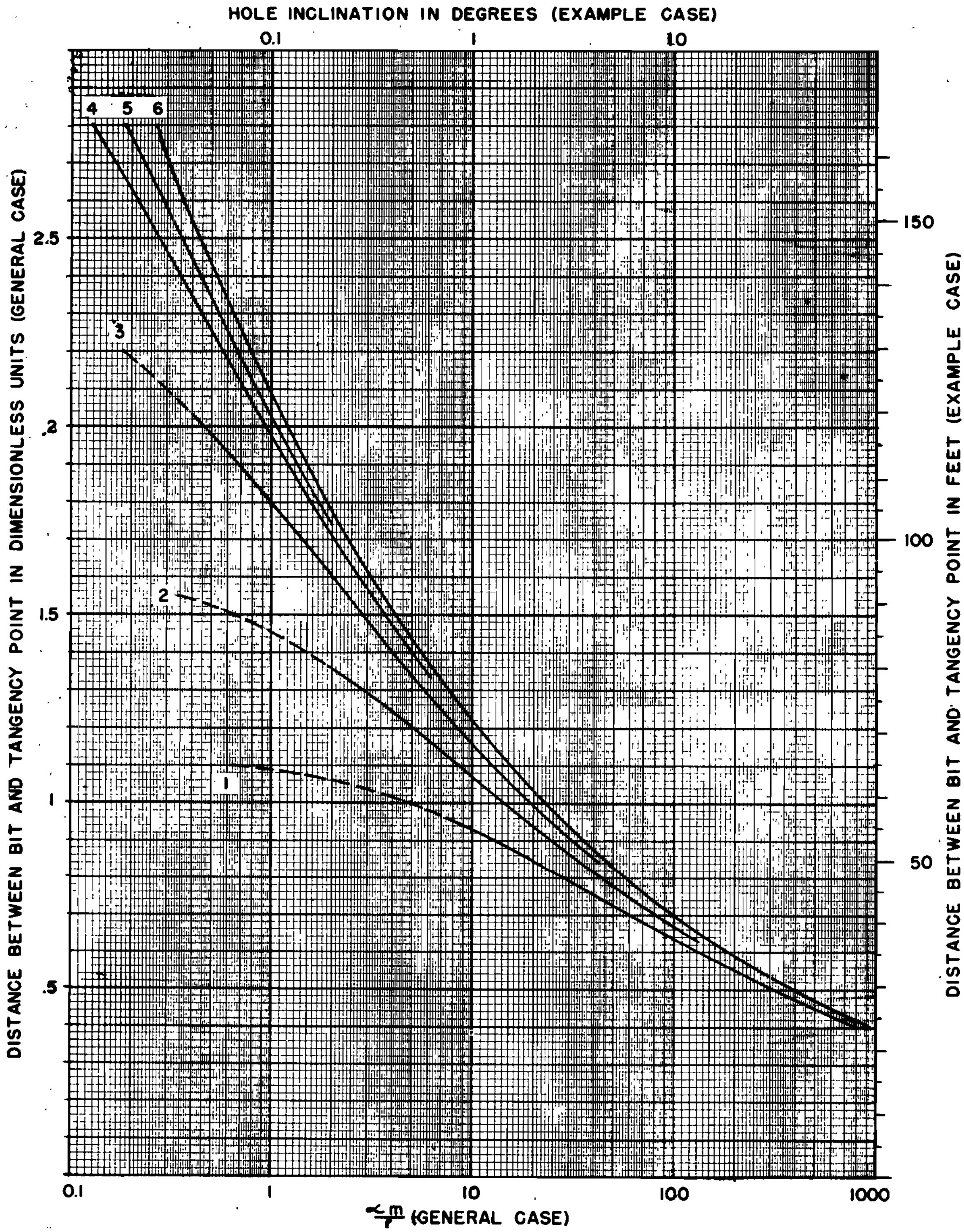

Fig. 5—Location of the Point of Tangency

	Curve 1	2	3	4	5	6
Example Case:						
6¼-in. drill collars, 8¾-in. hole, 10 lb per gal mud.						
Weight in pounds	36,000	18,000	9,000	5,400	4,500	3,600
General Case:						
Weight in dimensionless units	8	4	2	1.2	1	0.8

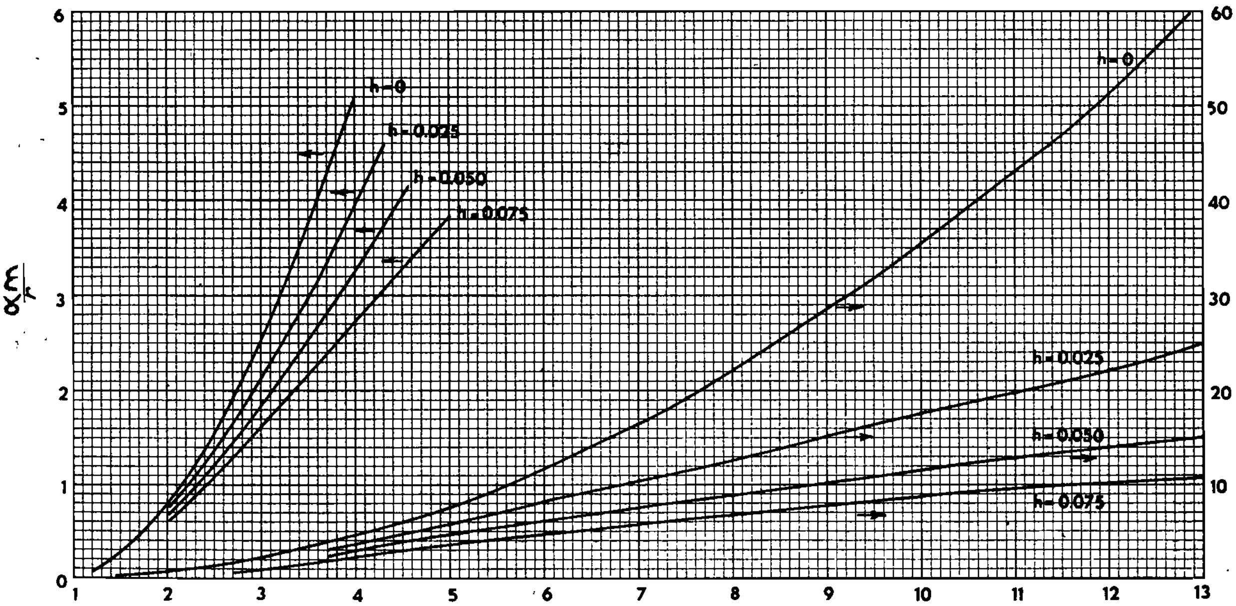

Fig. 6—Inclination Toward Which the Hole Tends (Equilibrium Angle) in Non-inclined Formations for Various Values of Anisotropy Index *h*
General Case

of drillabilities parallel and perpendicular to bedding planes will be referred to herein as the "drilling anisotropy index,"* and will be designated by the symbol h.

h equal to zero corresponds to an isotropic formation. As it will become apparent later herein, observed field phenomena are explained by the use of values of h between zero and 0.075.

HOLE INCLINATION IN ANISOTROPIC HORIZONTAL FORMATIONS

It was previously shown that the conditions for maintaining constant angle of hole inclination (equilibrium conditions) for isotropic formations were obtained by the intersection of the various curves of Fig. 2 with the horizontal line $\phi/\alpha = 1$. Similarly, as will be proved in the appendix, the equilibrium conditions for anisotropic† horizontal formations are represented by the points of intersection of the same curves with the horizontal line $\phi/\alpha = 1/1-h$. Consider, for instance, the case of $h = 0.05$. Then $\phi/\alpha = 1.0526$ (short dashed line in Fig. 2). For a weight of 8 dimensionless units (36,000 lb for 6¼-in. drill collars) the equilibrium conditions are represented by point G. As G is located to the left of B, the conclusion may be drawn that equilibrium angles are smaller in anisotropic horizontal formations than in isotropic formations.

* For more rigorous definition, see appendix.

† "Anisotropic" means having properties non-identical in all direction.

Values of the ratio $\alpha m/r$ for equilibrium conditions vs. the weight on the bit in dimensionless units were plotted in Fig. 6, in which various curves correspond to various values of the anisotropy index h.

Fig. 6 shows that hole inclination may be two or three times greater in isotropic formations $(h = 0)$ than in anisotropic horizontal formations.

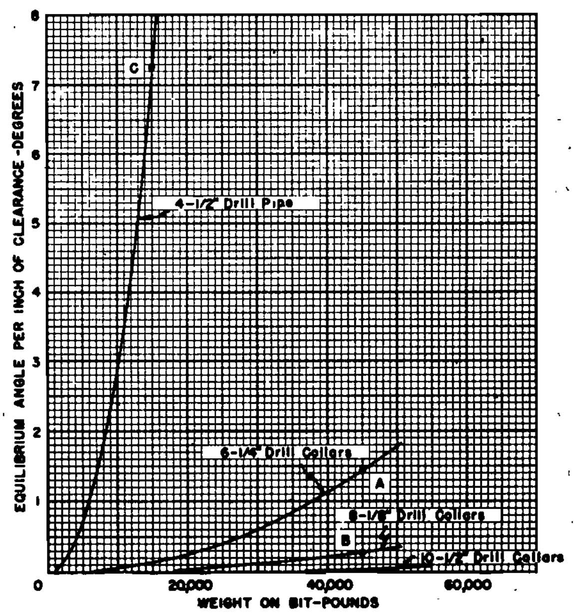

Fig. 7—Inclination Toward Which the Hole Tends (Equilibrium Angle) in Isotropic Formations
Numerical Examples

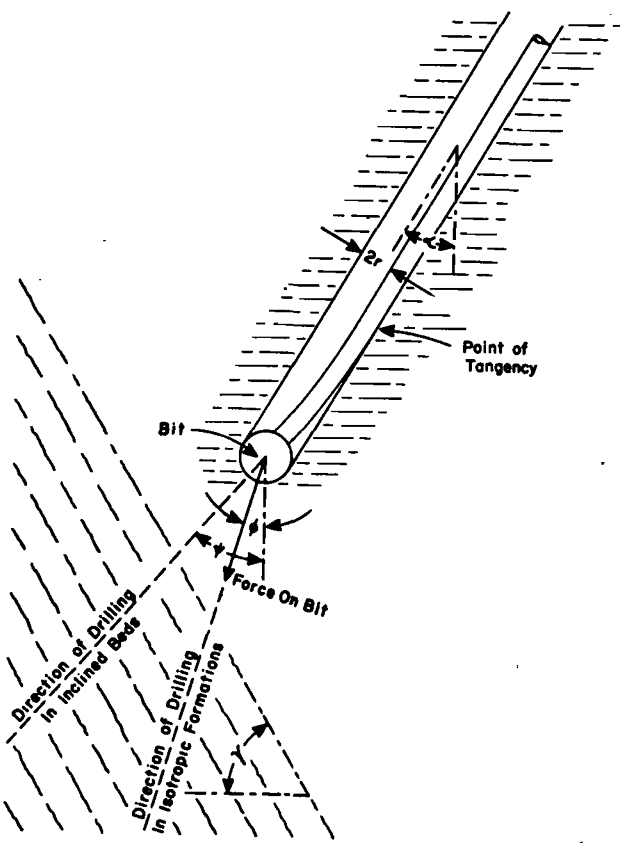

Fig. 8

Although the range of anisotropy indices in Fig. 6 was obtained from a consideration of situations in dipping formation, the results obtained from the use of the same range of values explain the low hole inclinations obtained in anisotropic horizontal beds such as in Kansas where heavy weights may be applied on short drill-collar strings.

HOLE INCLINATION IN ANISOTROPIC SLANTING FORMATIONS

Generalities

The determination of the angle toward which the hole tends, i. e., the equilibrium angle, for horizontal formations was relatively easy by use of Fig. 2. This problem is more complicated in the case of inclined formations, and for this reason its explanation will be given in the appendix. The results are plotted in Fig. 9, 10, and 11 for anisotropy indices h equal to 0.025, 0.050, and 0.075, respectively. The various drawings of each of these figures are for various formation dips, the dip of the formation being the angle γ of bedding plane with respect to the horizontal as shown in Fig. 8.

In all of these figures, the equilibrium angle in degrees is plotted vs. the diametral clearance in inches between the collars and the hole. Curves are given for various values of the weight on the bit in dimensionless units. A convenient conversion table of dimensionless units into pounds is given in Fig. 11.

The way of using Fig. 9 to 11 will be explained with a few examples.

Example 1

Consider, for instance, formations slanting 30 deg in which field practice indicates an angle of about 2½ deg will be maintained when drilling 9-in. hole with 6¼-in. drill collars and 4,500-lb weight. Hole clearance is then 2.75 in. and weight is one dimensionless unit, as indicated in the table of Fig. 11. Both Fig. 9d and 10d are for $\gamma = 30$ *deg*. Point A on Fig. 10d indicates an inclination of 2¾ deg, which is close to the actual field value; and consequently the anisotropy index h is equal to about 0.050.

We may now study the effect in this particular field of changes of weight, size of drill collars, and clearance.

If the weight is doubled to 2 dimensionless units or 9,000 lb, the hole inclination will increase to 6¼ deg as indicated by point B. If the weight is increased to 18,000 lb and 36,000 lb, the hole inclination will increase to 13¼ deg and 21½ deg, respectively, as indicated by points C and D.

Consider now that the 6¼-in. drill collars are replaced by 8⅛-in. drill collars. Point E (clearance = ⅞ in.) indicates that for one dimensionless unit of weight, which now means 9,000 lb, the inclination is 3½ deg. In other words, the use of larger drill collars and twice the weight resulted in only a small increase in inclination.

Consider now still larger drill collars, such as 10½-in. in 12¼-in. hole. Point F (clearance = 1¾ in.) indicates that 18,000-lb weight results in an inclination of 3 deg. In this case, however, the hole size was increased as well as the collar size, and the additional 9,000 lb are only partially used to drill faster and partially to drill a larger hole. A rough way of calculating the increase in drilling rate is to take the drilling rate as directly proportional to the weight per inch diameter of hole.* For example,

* Experiments show that the following relationship is quite accurate:

$$R = k\left(\frac{W}{D}\right)^n$$

wherein:

R is drilling rate. D is hole diameter.
W is weight on bit. k is a proportionality factor.
n is an exponent depending on the formation, which was taken equal to unity in the above approximation.

There is no experimental justification for the often-used relationship:

$$R = k\left(\frac{W}{D^2}\right)^n$$

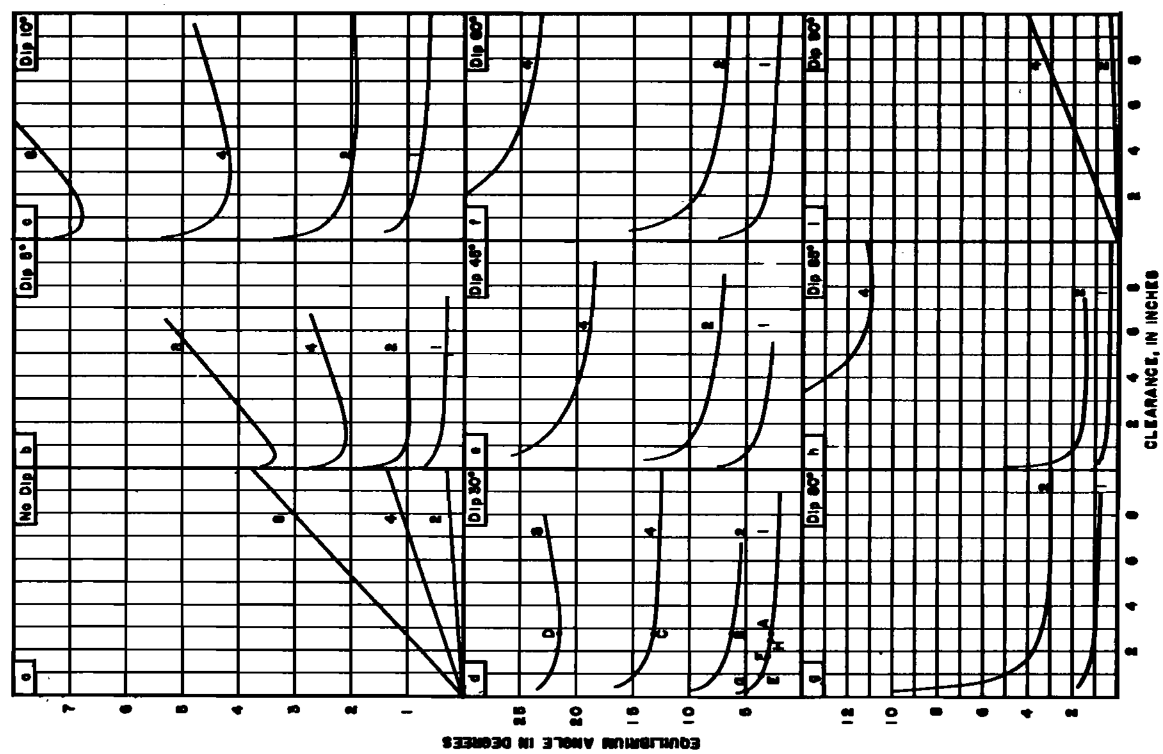

Fig. 10—Inclination Toward Which the Hole Tends (Equilibrium Angle) in Slanting Formations
Anisotropy Index $h = 0.050$.
Numbers on Curves Refer to Weight on Bit in Dimensionless Units (See Table, Fig. 11)

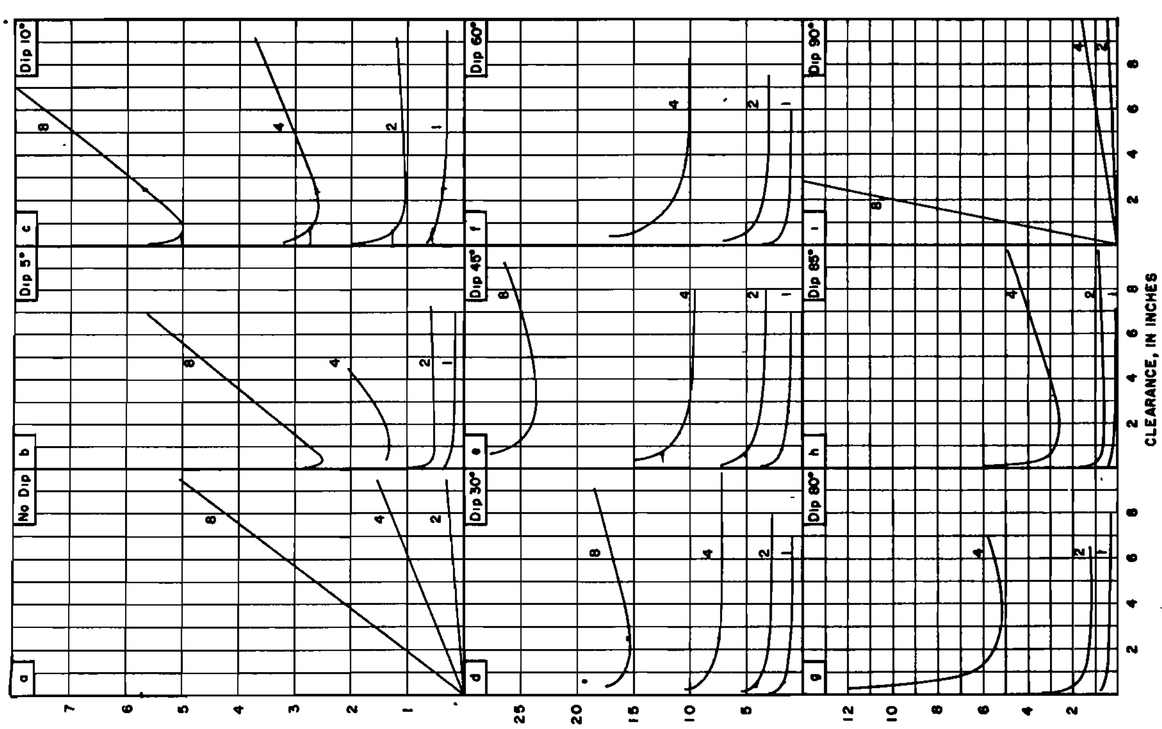

Fig. 9—Inclination Toward Which the Hole Tends (Equilibrium Angle) in Slanting Formations
Anisotropy Index $h = 0.025$.
Numbers on Curves Refer to Weight on Bit in Dimensionless Units (See Table, Fig. 11)

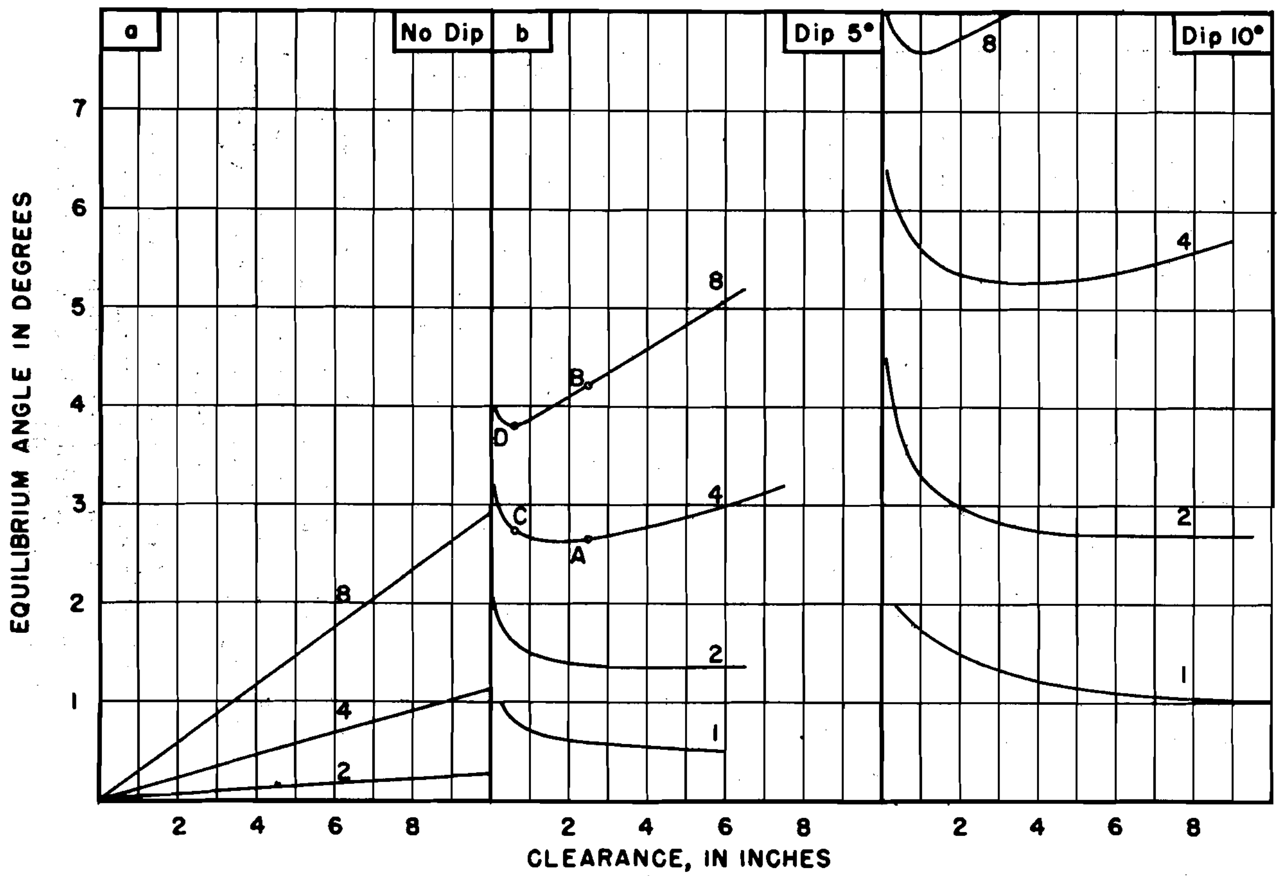

Fig. 11–Inclination Toward Which the Hole Tends (Equilibrium Angle) in Slanting Formations
Anisotropy Index $h = 0.075$. Numbers on Curves Refer to Weight on Bit in Dimensionless Units
Conversion of Dimensionless Units of Weight into Pounds

	Weight, Pounds			
Dimensionless Units:	1	2	4	8
$4\frac{1}{2}$-in. drill pipe	750	1,500	3,000	6,000
$6\frac{1}{4}$-in. drill collars	4,500	9,000	18,000	36,000
$8\frac{1}{8}$-in. drill collars	9,000	18,000	36,000	72,000
$10\frac{1}{2}$-in. drill collars	18,000	36,000	72,000	. . . . .

9,000 lb in 9-in. hole is 1,000 lb per inch; and 18,000 lb in $12\frac{1}{4}$-in. hole is 1,470 lb per inch. We would then expect the $12\frac{1}{4}$-in. bit with 18,000-lb weight to drill $1{,}470 \div 1{,}000 = 1.47$ times faster than a 9-in. bit with 9,000-lb weight. With this respect, better results would be obtained with $11\frac{1}{4}$-in. drill collars in $12\frac{1}{4}$-in. hole than with $10\frac{1}{2}$-in. drill collars in the same hole.

Finally, let us analyze the effect of changes of clearance. If 4,500-lb weight is carried on $6\frac{1}{4}$-in. drill collars in a $7\frac{7}{8}$-in. hole instead of a 9-in. hole, the inclination would be $3\frac{1}{2}$ deg (point E) instead of $2\frac{3}{4}$ deg (point A), which means that for less clearance the inclination is greater. The influence of the clearance would be greater for much smaller clearances such as $\frac{1}{4}$ in. (point G) for which the inclination would reach 5 deg. This finding, which, it should be remembered, is for beds inclined 30 deg, is quite unexpected.

The use of many stabilizers in the lower portion of a string of drill collars is approximately equivalent to a decrease of clearance between collars and hole without changing collar size. It appears then that in this example, in which the beds have a dip of 30 deg, the use of stabilizers would result in an increase of angle of inclination.*

The conditions of the foregoing example approximate fairly well drilling conditions in those areas

* It must be remembered that here we are considering only equilibrium angle of inclination. The advantage of stabilizers in preventing rapid changes of angle will be discussed further in this paper.

of southern Oklahoma, Wyoming, and elsewhere where the dip of the formations is between 25 and 45 deg.

The conclusion concerning the foregoing example is that for the conditions under consideration, there are two means of performing cheaper drilling, viz.:

1. Increase of the weight on the bit and deliberate drilling of more inclined holes (see A and B following). The advantage of the method in some wildcats is the generally increased probability of discovery because of the updip drift of the bit. On the other hand, the appreciable horizontal distance between surface and bottom location of wells may not be allowed by present state laws, or may be inconvenient in small leases. For large leases, it has already been proposed that surface locations may be made so as to bottom the wells as desired.[2]

2. Use of larger drill collars, which allow carrying more weight on the bit without increasing hole inclination (see C following). The authors of this paper recommend 8-in. or $8\frac{1}{8}$-in. drill collars in $8\frac{3}{4}$-in. and 9-in. holes, which are already being used. Furthermore, the use of still larger collars, such as $10\frac{1}{2}$-in. to $11\frac{1}{4}$-in. in $12\frac{1}{4}$-in. hole should be tried.

The foregoing two means, which may also be used jointly (see D following) are confirmed by the following field data:

A. In a paper presented in 1952,[3] R. B. McCloy reported savings of $10,000 to $30,000 obtained in Carter County, Oklahoma, by carrying more weight and allowing the well inclination to reach 7 deg.

B. Wesley W. Moore in the discussion of the same paper reported a well in Fremont County, Wyoming, in which the weight had to be maintained under 11,000 lb, with corresponding penetration rates of 22 to 24 ft per day, in order to keep the inclination below 5 deg. Later, it was learned from oriented cores that the course of the hole was updip; and, inasmuch as this was a wildcat well on a large lease, the weight was increased to 33,000 lb. The inclination reached 17 deg and the resulting rate of penetration became 75 ft per day. The resulting estimated saving in this 14,000-ft well was $85,000.

C. At a meeting of the API Mid-Continent Study Committee on Bore-hole Drift, W. M. Booth,[4] reported the drilling performance of many holes drilled in Sholem-Alechem Field of Oklahoma with $6\frac{1}{4}$-in. drill collars and 8-in. drill collars. With 10,000 to 20,000 lb and 8-in. drill collars, the same average inclination of $2\frac{1}{2}$ deg at the bottom of the hole was reached as with 2,000 to 10,000 lb and $6\frac{1}{4}$-in. drill collars. The large drill collars resulted in a saving of about one third in drilling time.

D. The writers of this paper know of a well in Wyoming in which 30,000 lb were carried with 8-in. drill collars in 9-in. hole. Fifteen degrees of inclination were reached. Conventional practice in the same field is to drill with $6\frac{1}{4}$-in. drill collars and 8,000-lb weight in order to keep the inclination small. The new method resulted in drilling progress of 100 ft per day, as compared to 35 to 40 ft per day by conventional methods.

Example 2

Let us now study another set of field conditions in which, with $6\frac{1}{4}$-in. drill collars in $8\frac{3}{4}$-in. hole, 5-deg angle of dip, and 20,000-lb weight, a 3-deg hole angle is maintained. These conditions are approximately met by point A, Fig. 11b, which indicates $2\frac{3}{4}$ deg inclination for 4 dimensionless units of weight (18,000 lb for $6\frac{1}{4}$-in. drill collars) and $2\frac{1}{2}$-in. clearance. If the weight were doubled to 36,000 lb, the hole angle would increase to $4\frac{1}{4}$ deg (point B). If, on the other hand, $8\frac{1}{8}$-in. drill collars were used instead of $6\frac{1}{4}$-in. and 36,000 lb carried, the hole angle would become $2\frac{3}{4}$ deg (point C). Finally, if 72,000 lb were carried on $8\frac{1}{8}$-in. collars, the hole angle would become $3\frac{3}{4}$ deg (point D). To investigate the effect of hole clearance alone, consider $6\frac{1}{4}$-in. drill collars with 36,000-lb weight and clearances equal to $2\frac{1}{2}$ in. (point B) and $\frac{5}{8}$ in. (point D) for which inclinations are $4\frac{1}{4}$ deg and $3\frac{3}{4}$ deg, respectively. This means that for less clearance the hole deviation is smaller, which is the reverse of the situation analyzed in the previous example for 30-deg dip. The effect of clearance on hole inclination increases as dip angle decreases. In the limiting case of zero dip, i. e. horizontal formations, Fig. 9a, 10a, and 11a show that the angle of inclination increases very fast with clearance and is, in fact, proportional to the clearance. For this reason, in the case of horizontal beds or isotropic formations previously investigated in this paper, inclinations per inch of clearance were given, as in Fig. 7. From inspection of curves of Fig. 11b, it is seen that the angle of inclination increases with the clearance only for the upper curve, i. e., for the highest weight.

The conditions of this last example correspond to crooked-hole territories in West Texas. W. S. Bachman and H. M. Rollins reported in a recent paper[5] the success attained by use of stabilizers in West Texas for weights as high as 50,000 lb on $8\frac{3}{4}$-in. bits.

Highly Inclined Beds

In Fig. 12 and 13 the angle toward which the hole tends, i.e., the equilibrium angle, is plotted vs. the formation dip for a clearance between the collars and the hole equal to 2.75 in. These figures are for anisotropy indices of 0.025 and 0.050, respectively.

We shall use these drawings to investigate the influence of very inclined beds.

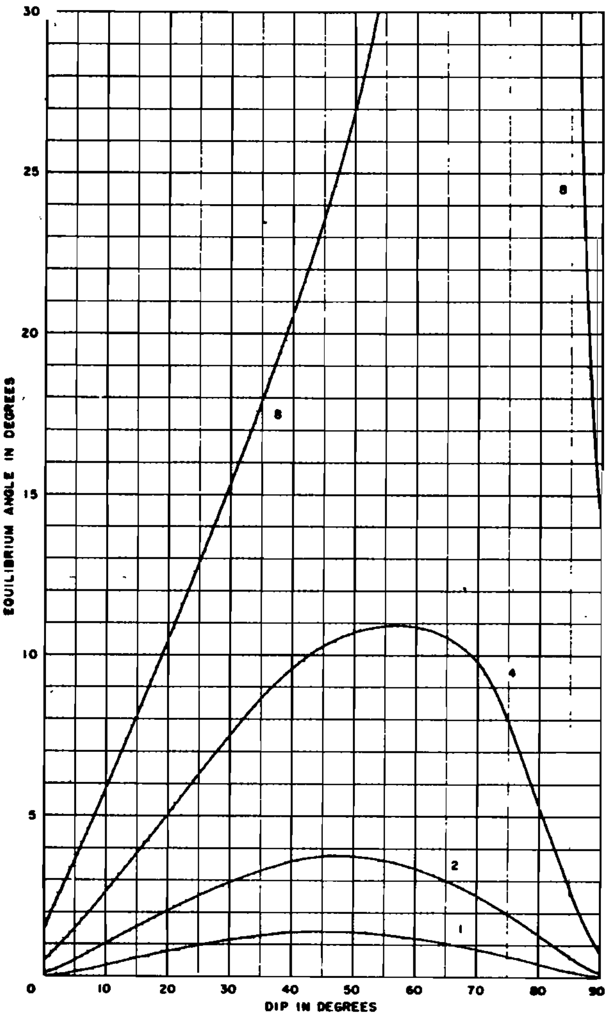

Fig. 12—Equilibrium Angle vs. Formation Dip
Anisotropy Index $h = 0.025$
Clearance = 2.75 in.
Numbers on Curves Refer to Weight on Bit in Dimensionless Units (See Table, Fig. 11)

Point A of Fig. 13 shows that for 20-deg dip, the inclination would be 14 deg for a weight on bit of 8 dimensionless units or 36,000 lb for 6¼-in. drill collars. Although high, this inclination is not unreasonable.

Following the curve for the same weight, we find that for 60-deg dip, the equilibrium angle is some value far off scale and altogether unacceptable. Point B shows that even with 4 dimensionless units of weight (18,000 lb for 6¼-in. collars) the hole angle is 28 deg. In such beds, only weights corresponding to one, two, or slightly more dimensionless units may be considered. The only possibility of carrying more weight without exceeding reasonable inclination is to use larger drill collars. The inclination obtained with 4,500-lb weight and 6¼-in. drill collars, 3 deg (point C) would also be obtained with 9,000 lb with 8⅛-in. collars or 18,000 lb with 10½-in. collars. Doubling these weights would result in an angle of 10 deg (point D).

It is interesting to note that for still steeper formations, the previously mentioned weight would lead to smaller angles of inclination; e.g., 1 deg (point E) and 3½ deg (point F), respectively, for an 80-deg dip. On the other hand, for larger values of weight the deviation would still be off scale.

As already stated, Fig. 13 is for a clearance of 2.75 in., which is the clearance for 6¼-in. collars in 9-in. holes. Small errors will be made when these charts are used for oversized collars. More correct values might be obtained by using Fig. 10.

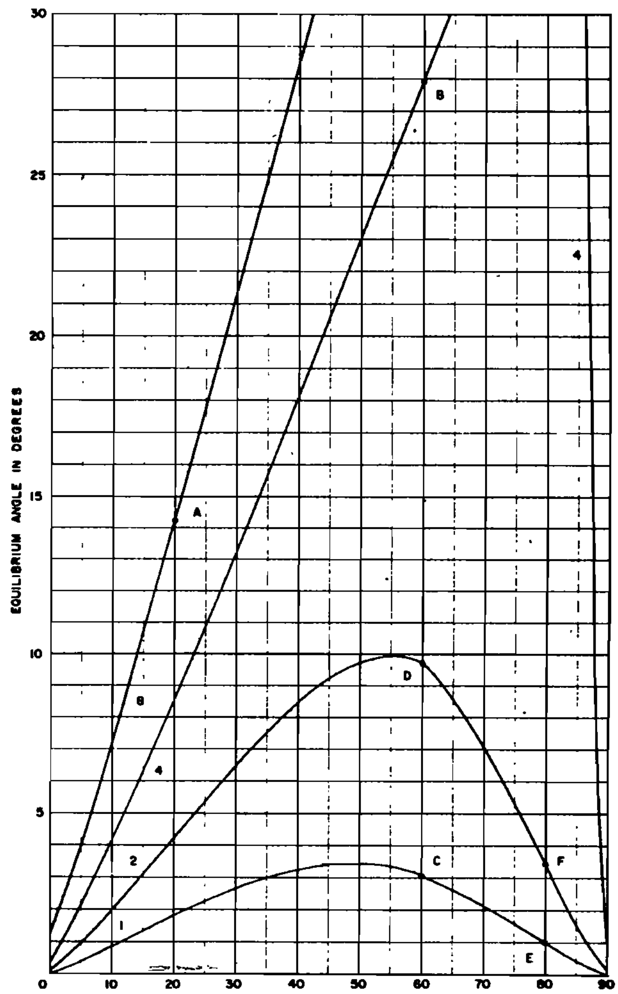

Fig. 13—Equilibrium Angle vs. Formation Dip
Anisotropy Index $h = 0.050$
Clearance = 2.75 in.
Numbers on Curves Refer to Weight on Bit in Dimensionless Units (See Table, Fig. 11)

Note Concerning Fig. 9, 10, and 11

The abscissa in Fig. 9, 10, and 11 is the clearance between the drill collars and the hole. In the examples, this abscissa was used for any size of collars. Actually, this scale is correct only for 6¼-in. collars; and, if used for 8⅛-in. or 10½-in. collars, the readings should be multiplied by 1.189 and 1.414, respectively. For instance, in example 1 the case of 18,000-lb weight, 10½-in. collars in 12¼-in. hole was analyzed, and point F (clearance 1¾ in.) of Fig. 10d was taken as representing these conditions. Actually, a clearance of 1¾ in. for 10½-in. collars corresponds to a clearance of $1.75 \times 1.414 = 2.475$ *in.* of the abscissa scale. Consequently, actual conditions are represented by point H instead of point F, which means that the hole inclination is 2¾ deg instead of 3 deg, which is a small error. In most cases, this error is still smaller; and, for this reason, a complication of providing Fig. 9, 10, and 11 with three abscissa scales for various collars was not deemed necessary.

Error for Large Hole Inclinations

The derivations in the appendix are based on the assumption that the hole inclination is small. This introduces an error in all of the curves. Although the magnitude of this error was not calculated, it is the authors' feeling that this error is negligible up to inclinations of 10 deg, small for 15 deg, and appreciable for 20 deg or more. For large hole inclinations, the actual deviations from vertical are smaller than those indicated in the curves.

DOG-LEGS

Generalities

Up to this point, we have been studying the angle toward which the hole tends, or equilibrium angle, with little or no consideration for the changes in hole angle which are taking place while this equilibrium is being reached. Actually, no troubles may be expected from the hole inclination itself. Directional holes are sometimes drilled with inclinations as high as 70 or 80 deg with a minimum of drilling or production troubles, even if beam pumped. On the other hand, troubles may be expected from dog-legs.

Dog-legs normally result from a change of conditions such as a change of weight or a change of formation property. Such changes cause an instantaneous change of hole direction, but the continuation of the hole along its new direction may soon be prevented by the drill collar contacting the wall of the hole close to the bit.

We shall first study the instantaneous change of angle as affected by: *1*, a sudden change of weight; *2*, a sudden change of formation dip, i.e., crossing an unconformity.

Next, we shall study the continuation of a dog-leg after the instantaneous change of angle occurs.

Start of a Dog-Leg by a Sudden Change of Weight

Sudden changes of angle caused by changes of weight were computed as explained in the appendix for a formation of anisotropy index $h = 0.025$, and plotted in Fig. 14 and 15 which are for a drop of weight from 36,000 to 18,000 lb and from 18,000 to 9,000 lb, respectively. In these drawings, the change of angle is plotted vs. the clearance between the drill collars and the hole. Curves are drawn for horizontal beds and for 30-deg dips, and for various sizes of drill collars.

It is immediately obvious from the inspection of these drawings that much higher instantaneous changes in angle result from a drop of weight in inclined beds than from the same drop of weight in horizontal beds; and that the effect of beds is much stronger than the size of drill collars.

A comparison of Fig. 14 and 15 shows that the order of magnitude of instantaneous change of angle is the same for a drop of weight from 36,000 to 18,000 lb as for a drop of weight from 18,000 to 9,000

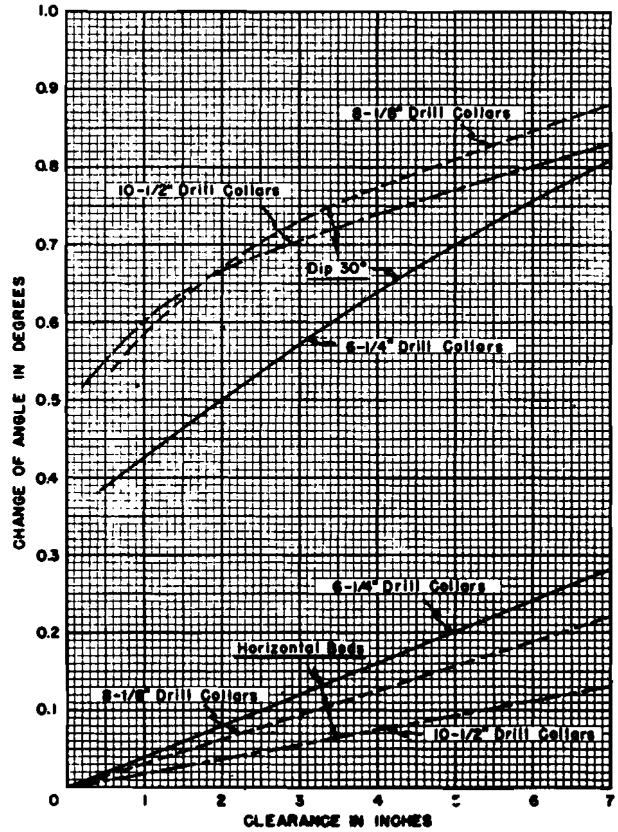

Fig. 14—Instantaneous Change of Angle Caused by Drop of Weight from 36,000 to 18,000 Lb
Anisotropy Index $h = 0.025$

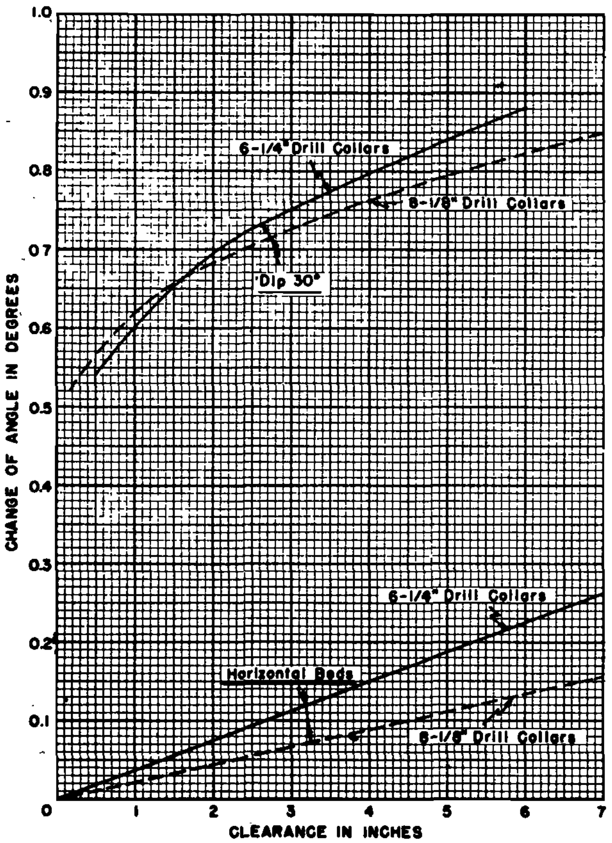

Fig. 15—Instantaneous Change of Angle Caused by Drop of Weight from 18,000 to 9,000 Lb
Anisotropy Index $h = 0.025$

lb, this change of angle being primarily influenced by the formation dip.

Thus, a conclusion may be reached that a recommendation previously made in this paper to drill cheaper holes by carrying more weight and accepting more deviation should not result in sharper dog-legs.

Both Fig. 14 and 15 show that for a 30-deg dip, the order of magnitude of the instantaneous change of angle is less than 1 deg. Although calculations have not been made for greater dips and greater anisotropy indices, it is apparent to the authors that for such conditions the change of angle would be more than proportionally greater. This indicates that in highly inclined beds, sudden changes of weight should be avoided.

Fig. 14 shows that 6¼-in. collars give lower angle changes than larger collars for 30-deg dips when the weight is dropped from 36,000 to 18,000 lb. For other dips, opposite results might be obtained and actually no conclusions can be reached about the influence of collar size on the instantaneous change of hole angle in dipping formations.

Consider the effect of hole clearance on the instantaneous change of angle caused by a sudden drop of weight. Both Fig. 14 and 15 show that, in all cases, reducing the clearance reduces the change of angle. Percentagewise, this effect is much smaller in inclined beds than in horizontal beds. Consequently, it seems that reducing the clearance itself, or using many stabilizers in the lower portion of the string, would be of doubtful benefit in reducing the instantaneous change of angle in steeply dipping formations but may be of great benefit in slightly dipping formations.

In the foregoing study, we considered effects of a sudden drop in weight. A sudden increase in weight gives similar results.

Start of a Dog-Leg by Crossing Unconformity

Suppose that, while drilling, an unconformity is encountered and that the beds are horizontal on one side and inclined on the other. Then, as proved in the appendix, the instantaneous change of angle $\psi-\alpha$ (see Fig. 8) is equal to:

$$\psi-\alpha = h\gamma$$

in which h is the anisotropy index and γ the formation dip on one side of the unconformity. This formula means that the instantaneous change of angle does not depend upon weight, collar size, clearance, or hole inclination, but depends only upon formation. Thus, once more we conclude that the previous recommendation of drilling with more weight on the bit and letting the inclination build up does not result in sharper dog-legs.

Take, for instance, $h = 0.025$ and $\gamma = 30\,deg$. Then, the instantaneous change of angle is 0.75 deg. The change in angle will be greater for steeper dips and for higher values of anisotropy index.

Actually, the foregoing formula holds true only for small dips. For large dips, the instantaneous change of angle depends not only upon the formation, but also upon the other previously mentioned factors. The authors believe, however, that for dips such as 30 deg, the formula gives a satisfactory approximation.

Continuation of a Dog-Leg

So far we have investigated the instantaneous change of angle, which is the start of a dog-leg. Let us follow the continuation of a dog-leg. Fig. 16a shows a straight, but inclined, hole drilled through steeply inclined formations to an unconformity on the other side of which the bedding planes are horizontal. As the horizontal formations are penetrated, there is an instantaneous change of angle. Drilling progresses as indicated by the dashed lines in Fig. 16a until the condition represented in Fig. 16b is reached. At this stage, the drill collar contacts the

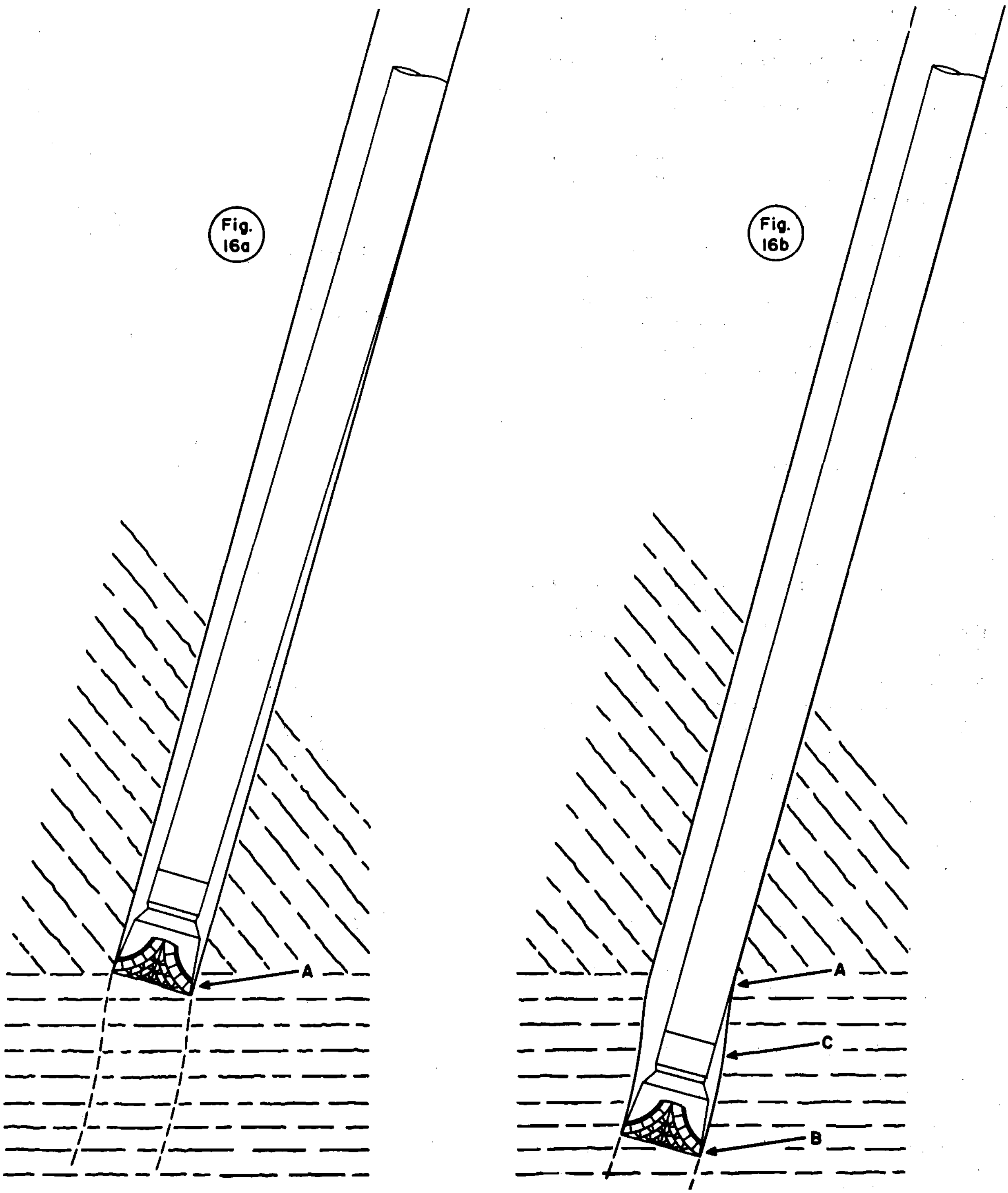

Fig. 16—Dog-Leg Caused by Going from Inclined to Horizontal Beds

wall of the hole at the unconformity (point A). When drilling below point B, as indicated by dashed lines in Fig. 16b, the bit will drill parallel to the original hole until it has advanced far enough for the overhanging portion of the drill collar below point A to impart to the bit an appreciable force toward vertical.

If the clearance between collar and hole were smaller, the instantaneous change of angle would be substantially the same, as previously explained. However, the drill collar would then contact the wall at point A when the bit had advanced to some point above B, such as C. The hole would then continue from point C in a direction parallel to the original hole. It is obvious that the reduction of clearance reduces the misalignment between the upper and lower parallel portions of the hole. This misalignment is equal to one-half the clearance between the collar and hole. Thus, the reduction of clearance reduces the severity of dog-leg.

In Fig. 16, the dog-leg is directed toward vertical. The foregoing conclusion holds true, however, for any dog-leg.

It is the authors' belief that, for steeply inclined formations, this reduction of severity of dog-legs is the main advantage in the use of smaller hole to collar clearances or, what is more or less equivalent, in the use of several stabilizers in the lower portion of the string.

In Fig. 16 the dog-leg was produced by an unconformity. A similar dog-leg would be produced if the weight were suddenly dropped, as is often done when it is desired to reduce deviation. The resulting hole is illustrated by the solid lines in Fig. 17. If the weight were gradually reduced to its final value, a hole illustrated by the dashed lines would be drilled. The results would be: *1*, the same overall hole straightening; 2, no dog-leg; 3, faster drilling.

When it is desired to increase the weight on the bit, a gradual increase will also avoid a dog-leg.

The conclusion is that in crooked-hole territory, large changes in weight should be graduated over a 15- to 30-ft interval.

HELICAL BUCKLING

This investigation was made with an assumption that the drill collars lie on the low side of the hole, as shown in Fig. 1. Actually, even in the absence of any rotation, there are conditions for which the string does not lie on the low side of the hole, but buckles into a helix.

A theoretical study shows that, for a given dimensionless weight on the bit, there is a value of $\propto m/r$ below which helical buckling may occur and above

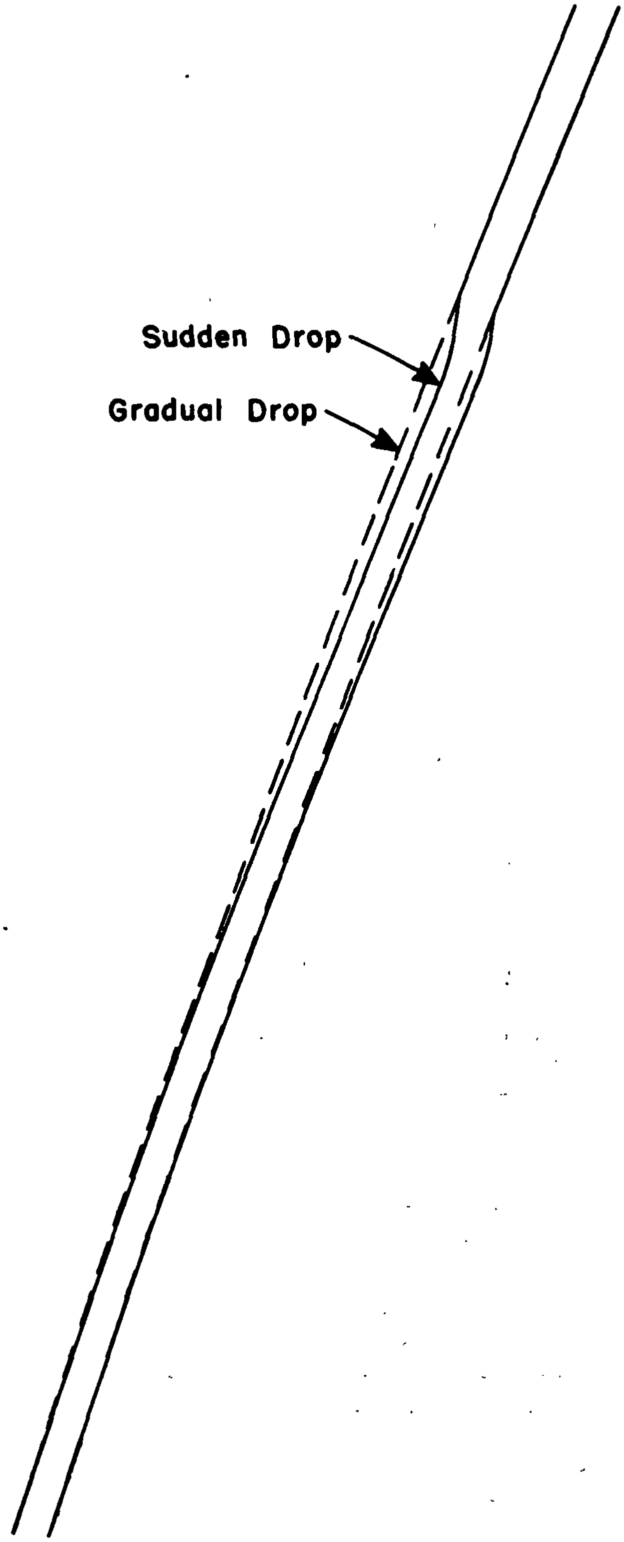

Fig. 17—Effect of Drop of Weight

which it cannot occur. Model experiments were made fixing the limiting value of $\alpha m/r$ for different dimensionless weights. The results are shown in Fig. 2 and 5 in which the portions of the curves drawn with dashed lines indicate helical buckling conditions.

The use of the curves of Fig. 2 has shown that helical buckling occurs only when the weight is suddenly increased in a nearly vertical hole drilled in horizontal or nearly horizontal formations. In inclined beds the collars always lie on the low side of the hole. In horizontal beds the collars lie on the low side of the hole for equilibrium conditions and when the weight is being decreased. In fact, in almost any actual situation encountered in drilling, the possibility of helical buckling of drill collars may be ignored.

SUGGESTIONS FOR FUTURE RESEARCH

It was assumed in this paper that, in isotropic formations, drilling is performed in the direction of the force imparted to the bit. In other words, it was implicitly assumed that bit ability to drill is the same in the axial and lateral directions. Actually, these two abilities are not necessarily equal and their ratio certainly depends upon the type of bit. An extensive experimental program is being conducted now by one of the authors in order to determine the relative abilities of bits to drill laterally and axially. After completion of this program, the mathematical findings of this paper might require some modifications to make allowance for the type of bit.

The use of stabilizers investigated in this paper concerns the case of many stabilizers located in the lower portion of the string. The effect of the location of one stabilizer only could be studied by the means used in this investigation. A previous publication[6] concerns the location of one stabilizer in a vertical hole and not in an inclined hole. Similarly, the effect of a limber joint could be analyzed.

The downdip drift of bore holes was not mentioned in this paper. It is well known that in very steep and soft beds the bit may drift downdip or along a contour line. The authors believe that these phenomena might be studied by making allowance for frictional rotational effects and for differences in the ability of the bit to drill laterally and axially.

A similar mathematical study without the simplifying assumption of small hole inclinations might be conducted. The results would not only provide a correction for the most inclined holes investigated in this paper, but would also extend the findings to the entire range of conditions encountered in directional drilling.

CONCLUSIONS

1. Drilling a perfectly vertical hole with an elastic drilling string is impossible, even in horizontal formations, unless very low and uneconomical weights are carried.

2. In crooked-hole territories of hard-rock areas, such as West Texas, Mid-Continent, and Rocky Mountains, cost of drilling may be lowered by:
   a. Carrying much more weight and deliberately accepting a larger deviation;* this does not result in sharper dog-legs. Such a method could not be used in steeply dipping formations because unreasonable deviation would result.
   b. Using larger drill collars with which more weight may be carried without increasing deviation. Examples: 8-in. collars in 9-in. hole, 11-in. collars in 12¼-in. hole.
   c. Using both foregoing methods in conjunction.

3. Extreme reduction in hole clearance, such as by the use of several stabilizers in the lower part of the collar string results in:
   a. A decrease in severity of dog-legs.
   b. A decrease in hole inclination in slightly dipping formations for heavy weights only.
   c. No decrease in hole inclination in steeply dipping formations.

4. Sudden large changes of weight in crooked-hole territories cause dog-legs and should be avoided. Any large change of weight should be graduated over a 15- to 30-ft interval.

APPENDIX

DIRECTION OF THE FORCE ON BIT

Differential Equation Method

The coordinate axes, abbreviations, general solution of the differential equation, and designations, if not defined below, are the same as in the appendix of ref. 1.

Consider a straight, but inclined, hole, the low side of which is represented by the straight line DC in Fig. 18. The angle of inclination with respect to the vertical is α. The curve AB represents the elastic line of the drilling string. A is the point of tangency and B is the bit. Let x_2 and x_3 designate the distances in dimensionless units from the neutral point to the bit and the tangency point, respectively.

The differential equation of this string is the same as the differential equation of the string in a vertical hole investigated in ref. 1, on condition that the angle α is small.

It results from the inspection of Fig. 18 that the difference of deflections between points A and B is equal to:

$$\frac{r}{m} + (x_2 - x_3)\,\alpha \text{ dimensionless units.}$$

* Large savings were made by carrying more than 30,000 lb and letting holes go off as much as 17 deg. In wildcats, the updip drift may increase the chances of a discovery.

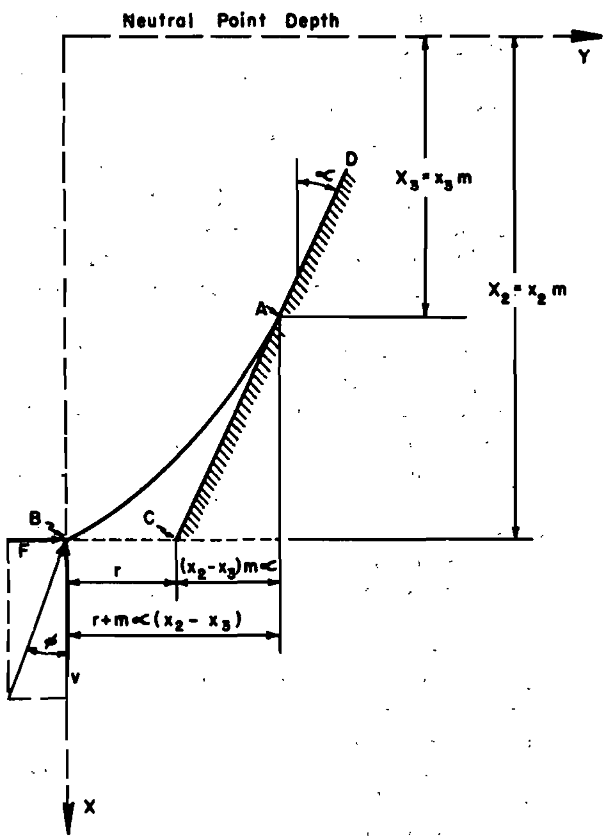

Fig. 18—Differential Equation Method

This fact is expressed by equation (1) following.

Equations (2) and (3) following express the fact that the bending moment is nil at points A and B, respectively.

Finally, equation (4) following expresses the fact that the drilling string is tangent to the hole at A.

$$\left\{\begin{array}{llll} a(S_3-S_2)+b(T_3-T_2)+c(U_3-U_2) & & & =\left[\frac{r}{m}\right]+(x_2-x_3)\propto \quad (1)\\ aP_3 & +bQ_3 & +cR_3 & =0 \quad (2)\\ aP_2 & +bQ_2 & +cR_2 & =0 \quad (3)\\ aF_3 & +bG_3 & +cH_3 & =-\propto \quad (4)\end{array}\right.$$

Divide a, b, c, and the second members of the equations by $\propto$.

Eliminating $a/\propto$, $b/\propto$, and $c/\propto$ between the four equations, we obtain:

$$\begin{vmatrix} S_3-S_2 & T_3-T_2 & U_3-U_2 & \left[\frac{r}{\propto m}\right]+(x_2-x_3) \\ P_3 & Q_3 & R_3 & 0 \\ P_2 & Q_2 & R_2 & 0 \\ F_3 & G_3 & H_3 & -1 \end{vmatrix} = 0 \qquad (5)$$

Solving (5) for $\propto m/r$, numerical values of $\propto m/r$ were computed for various values of x_2 (dimensionless weight on the bit) and x_3 (dimensionless distance from neutral point to tangency point). x_2-x_3 was plotted vs. $\propto m/r$ in Fig. 5. Each curve is for a constant value of x_2.

The expression of the inclination ϕ of the force on bit is:*

$$\phi = \frac{c}{x_2}$$

or:

$$\frac{\phi}{\propto} = \left(\frac{c}{\propto}\right)\left(\frac{1}{x_2}\right) \qquad (6)$$

Let us solve the set of equations (2), (3), and (4) for $c/\propto$. Then, for various numerical values of x_2 and x_3, corresponding values of $c/\propto$ are calculated. Substituting into (6) corresponding values of $\phi/\propto$ are obtained. Thus, both $\phi/\propto$ and $\propto m/r$ are known for various sets of values of x_2 and x_3. $\phi/\propto$ was plotted vs. $\propto m/r$ in Fig. 2. Each curve is for a constant value of x_2.

Iteration Method

The functions which are particular solutions of the differential equation were computed in ref. 1 by series for the values of x up to $x = 4.218$ only. In this investigation, greater values of x for which convergence of series is very poor, had to be considered. For this reason the following iteration method was devised.

We shall choose, as axes of coordinates, X and Y as shown in Fig. 19 and denote by X_1 the distance from the bit to the point of tangency. The following equation represents a curve passing through B and A (see Fig. 19) and satisfying the boundary conditions at these points. These boundary conditions are as follows: deflections equal to zero and $r+\propto X_1$ at B and A, respectively; $d^2Y/dX^2=0$ at B and A; curve tangent at A to the straight line CD.

$$Y = \frac{r}{\pi}\sin\frac{\pi X}{X_1} + (r+\propto X_1)\frac{X}{X_1} \qquad (7)$$

If this curve were the elastic line, then the bending moment at any point P would be (see Fig. 19):

$$EI\frac{d^2Y}{dX^2} = FX - VY + \int_0^X (Y-\eta)\,p\,d\xi \qquad (8)$$

wherein: p is the weight of string per unit length; and ξ and η are the coordinates of any point Q between B and P. The relationship between η and ξ is the same as the relationship (7) between Y and X. Substituting the expressions of Y and η into (8) and integrating, we obtain:

$$EI\frac{d^2Y}{dX^2} = FX - \frac{Vr}{\pi}\sin\frac{\pi X}{X_1} - \frac{Vr\lambda}{X_1} - V\propto X + \frac{pr}{\pi}X\sin\frac{\pi X}{X_1} + \frac{prX^2}{2X_1} + \frac{p\propto X^2}{2} + \frac{prX_1}{\pi^2}\left[\cos\frac{\pi X}{X_1} - 1\right] \qquad (9)$$

* With other symbols, this expression on page 208 of ref. 1 was: $\beta = c_2/x_2$.

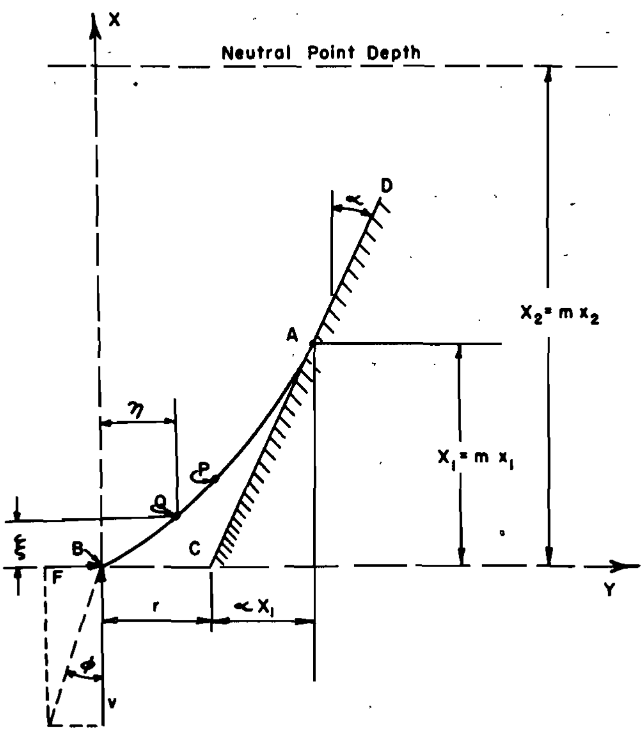

Fig. 19—Iteration Method

Expressing the fact that the bending moment is nil at A, i. e., making $d^2Y/dX^2=0$ and $X = X_1$ in (9), making the following substitutions:

$$F = V\phi \tag{10}$$

$$V = pm\,x_2 \tag{11}$$

$$X_1 = m\,x_1 \tag{12}$$

and rearranging, we obtain:

$$\frac{\phi}{\alpha} = \frac{r}{\alpha m}\left[\frac{1}{x_1} - \frac{1}{x_2}\left(\frac{1}{2} - \frac{2}{\pi^2}\right)\right] + 1 - \frac{x_1}{2\,x_2} \tag{13}$$

Integrating (9) and determining the integration constant with the condition $dY/dX=\alpha$ for $X=X_1$ (point A), an expression is obtained. Integrating this last expression and determining the integration constant with the condition $Y = 0$ for $X = 0$ (point B) another expression is obtained. Then, making in this last expression $Y = r + \alpha X_1$ and $X = X_1$ (point A), a relationship is obtained which, after substitution of (10), (11), and (12) and after rearranging, becomes:

$$\frac{\phi}{\alpha} = \frac{r}{\alpha m}\left[\frac{1}{x_1}\left(1+\frac{3}{\pi^2}\right) + \frac{1}{x_2}\left(\frac{18}{\pi^4} - \frac{3}{2\pi^2} - \frac{3}{8}\right) - \frac{3}{x_1 x_2^3}\right] + 1 - \left[\frac{3}{8}\right]\left[\frac{x_1}{x_2}\right] \tag{14}$$

Eliminating ϕ/α between (13) and (14) and solving for $r/\alpha m$:

$$\frac{r}{\alpha m} = \frac{x_1}{\dfrac{24}{x_1^3} - \left[\dfrac{24}{\pi^2}\right]\left[\dfrac{x_2}{x_1}\right] + \left(\dfrac{28}{\pi^2} - \dfrac{144}{\pi^4} - 1\right)} \tag{15}$$

This equation is similar to equation (5) of the differential equation method. It gives $r/\alpha m$ as a function of the dimensionless weight on bit x_2 and the distance x_1 from the bit to the tangency point in dimensionless units. Then, substituting (15) into (13), ϕ/α may be computed.

The results obtained with the iteration method and the differential equation method are identical. Small differences appear only in the region of helical buckling (dashed portion of curves in Fig. 2 and 5) which are of no interest. This proves that there is no need for a second iteration.

Consider now equilibrium conditions in isotropic formations. Making $\phi/\alpha = 1$ in (13) and eliminating x_1 between (13) and (15), a relationship between $\alpha m/r$ and x_2 was obtained and plotted in Fig. 6 (curve $h = 0$). Because an algebraic elimination of x_1 between (13) and (15) was not easy, the actual operations were performed as follows: First, $r/\alpha m$ was eliminated between (13) and (15) and the resulting equation solved for x_2. Then, values of x_2 were computed for many arbitrarily chosen values of x_1. Finally, corresponding values of $\alpha m/r$ were obtained with (15).

HOLE INCLINATION IN ANISOTROPIC FORMATIONS

Consider Fig. 20, in which V and F represent the vertical and horizontal components, respectively, of the reaction of the bottom of the hole on the bit and in which the straight line d is the direction perpendicular to bedding planes and making an angle γ with the vertical.

Projecting the reaction of the bottom of the hole on the axis d and on an axis e perpendicular to d (see Fig. 20), we obtain:

Component perpendicular to the bedding planes

$$F \sin\gamma + V \cos\gamma$$

Component parallel to the bedding planes

$$F \cos\gamma - V \sin\gamma$$

We may write that the component of an instantaneous displacement in the direction perpendicular to the bedding planes, i. e., along axis d, is:

$$n\,(F \sin\gamma + V \cos\gamma)$$

wherein: n is a proportionality factor.

Assume that the component of displacement in the direction e is smaller for anistropic formations than

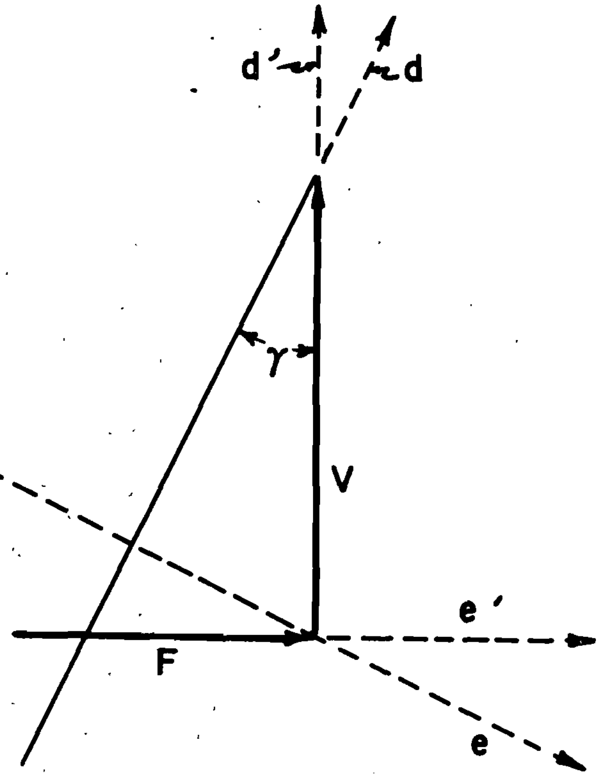

Fig. 20

for isotropic formations. Then, this component may be written:

$$nh'(F \cos \gamma - V \sin \gamma)$$

wherein: h' is a positive factor smaller than unity.

By definition, the anisotropy index h is equal to:

$$h = 1 - h' \qquad (16)$$

Projecting the above two components of displacement on a vertical axis d' and a horizontal axis e', we obtain:

Vertical component of displacement:

$n(F \sin\gamma + V \cos \gamma) \cos \gamma - nh'(F \cos \gamma - V \sin \gamma) \sin\gamma$

Horizontal component of displacement:

$n(F \sin\gamma + V \cos \gamma) \sin\gamma + nh'(F \cos \gamma - V \sin \gamma) \cos \gamma$

Consequently, if ψ is the inclination of the direction of displacement, we may write that $\tan \psi$ is equal to the ratio of the horizontal by the vertical components. Then, assuming ψ small and replacing $\tan \psi$ by ψ, substituting $V\phi$ for F and rearranging, we obtain:

$$\psi = \frac{\left(\frac{\phi}{\alpha}\right) \alpha (1 - h \cos^2 \gamma) + h \sin\gamma \cos\gamma}{\left(\frac{\phi}{\alpha}\right) \alpha h \sin \gamma \cos \gamma + (1 - h \sin^2 \gamma)} \qquad (17)$$

and for equilibrium conditions ($\psi = \alpha$):

$$\alpha = \frac{\left(\frac{\phi}{\alpha}\right) \alpha (1 - h \cos^2 \gamma) + h \sin\gamma \cos\gamma}{\left(\frac{\phi}{\alpha}\right) \alpha h \sin\gamma \cos\gamma + (1 - h \sin^2 \gamma)} \qquad (18)$$

Anisotropic Horizontal Formations

For anisotropic horizontal formations $\gamma = 0$ and, consequently, formula (18) for equilibrium conditions becomes:

$$\frac{\phi}{\alpha} = \frac{1}{1-h} \qquad (19)$$

instead of $\phi/\alpha = 1$ for isotropic formations. The relationship between $\alpha m/r$ and x_2 for various values of h was computed by the method already explained for isotropic formations in the section "Iteration Method" and plotted in Fig. 6.

Anisotropic Slanting Formations

We shall explain how the data of Fig. 9, 10, and 11 were obtained. For any given curve, the anisotropy index h, the inclination of beds γ, and the dimensionless weight on bit x_2 are constant and known; and, therefore, the variables in equation (18) are ϕ/α and α.

Let us arbitrarily choose a few values of $\alpha\ m/r$. Corresponding values of ϕ/α are obtained from Fig. 2. Then, equation (18) contains only the variable α, for which it is solved.

Thus, for each arbitrarily chosen value of $\alpha m/r$, a corresponding value of α was obtained; and finally, dividing the second by the first, a corresponding value of r/m was calculated. If Fig. 9, 10, and 11 were rigorous, the abscissa should be r/m and not the clearance $2r$. With this last abscissa, Fig. 9, 10, and 11 are correct only for the value of m corresponding to 6¼-in. drill collars and approximate for other collars.

In order to ascertain that the equilibrium conditions of Fig. 9, 10, and 11 are stable, formula (17) was transformed as follows:

$$\psi = \frac{\left(\frac{\phi}{\alpha}\right)\left(\frac{\alpha m}{r}\right)(1 - h \cos^2 \gamma) + \frac{m}{r} h \sin\gamma \cos\gamma}{\left(\frac{\phi}{\alpha}\right)\left(\frac{\alpha m}{r}\right) h \sin\gamma \cos\gamma + \frac{m}{r}(1 - h \sin^2 \gamma)} \qquad (20)$$

Assume, as above, that values of h, γ, and x_2 are given and constant. Assume further a given size of drill collars and hole. Consequently, m/r is also known and constant. Let us arbitrarily choose values of $\alpha m/r$. Corresponding values of ϕ/α are obtained from Fig. 2 and those of ψ/α are calculated with formula (20). By such numerical computations, it was found that for equilibrium conditions, i. e., for $\psi/\alpha = 1$:

$$\frac{\partial \frac{\psi}{\alpha}}{\partial \frac{\alpha m}{r}} < 0$$

which means stable equilibrium.

The curves of Fig. 9, 10, and 11 show that the equilibrium angle is minimum for some value of clearance, which, in most cases, is within the range of actual conditions. For less clearance than the foregoing value, the equilibrium angle becomes greater. This surprising fact was described in the body of this paper. An analysis is deemed necessary here in order to show that a proof may be given, which is independent from the arbitrary concept of drilling anisotropy* introduced in this investigation.

In a slanting formation the force on the bit is in a direction which tends to bring the hole toward vertical; and, consequently, $\phi/\propto$ is less than unity. In other words, the conditions are represented in Fig. 2 by a point such as H. Then, consider the conditions represented by point K, for which the weight on the bit and $\phi/\propto$ are the same as previously. Assume that the hole inclination $\propto$ is the same. Consequently, the inclination ϕ of the force on the bit is also identical in both cases. The magnitude and the direction of the force on the bit and the formation being the same, we may conclude that in both cases the equilibrium angle must be the same. Inasmuch as values of $\propto$ were assumed equal in both cases and as values of $\propto m/r$ are different at H and K, therefore, values of r are not equal. Consequently, there are generally two values of the apparent radius of the hole for which the equilibrium angle is the same, which explains the shape of curves of Fig. 9, 10, and 11.

Slightly Slanting Formations

For small γ, substitute in (17) γ for $\sin\gamma$ and unity for $\cos\gamma$ and disregard $h\gamma^2$ and $\phi h\gamma$, both small compared to unity.

$$\frac{\psi}{\propto} = (1-h)\frac{\phi}{\propto} + h\frac{\gamma}{\propto} \qquad (21)$$

and for equilibrium conditions ($\psi = \propto$):

$$\propto = \frac{h}{1-(1-h)\dfrac{\phi}{\propto}}\gamma \qquad (22)$$

Formulae (21) and (22) were used for formations inclined 5 and 10 deg.

Downdip Drift

We have assumed so far an updip drift of the bore hole. For a downdip drift, γ should be considered negative in formulae (17), (18), (20), (21), and (22). Numerical analysis of these formulae indicates that downdip stable equilibrium conditions may occur only for γ close to zero or to 90 deg. For small dips, downdip drift is impossible for a weight on the bit of one dimensionless unit or less. The greater the weight on the bit, the wider the range of γ for which downdip drift is possible. For 8 dimensionless units of weight, downdrift is possible when the formations are inclined less than 4 deg for a conventional amount of hole clearance and less than 2 deg for small clearances.

Downdip drift for slightly slanting formations is not a very important phenomenon and, for this reason, was not treated in the body of this paper. The main purpose of explaining it here is its application to updip drift in nearly vertical formations.

Nearly Vertical Formations

The straight line e in Fig. 21 represents a direction parallel to nearly vertical formations. The straight line d is perpendicular to e. The straight line a represents the axis of an updip hole.

Analyzing the means by which the drilling anisotropy concept was introduced in this investigation, it is apparent that drilling conditions would remain the same if the bedding planes were parallel to d instead of e, and the factor h' [see formula (16)] were replaced by its reciprocal. On the other hand, it results from an inspection of Fig. 21 that the updip drift of the axis a of the hole with respect to bedding planes e corresponds to a downdip drift with respect to d. Consequently, we must replace in our consideration γ by $-(90\text{ deg}-\gamma)$. Consider, for instance, the case of $\gamma = 85$ *deg.*, $h = 0.050$; i.e., $h' = 0.950$. Then, instead of an 85-deg dip, we may consider a dip of -5 deg and h' equal to $1/0.950=1.0526$; i.e., $h = 1-h' = -0.0526$. This method permits use of simple formulae (21) and (22), instead of more complicated expressions (17) and (18) in the case

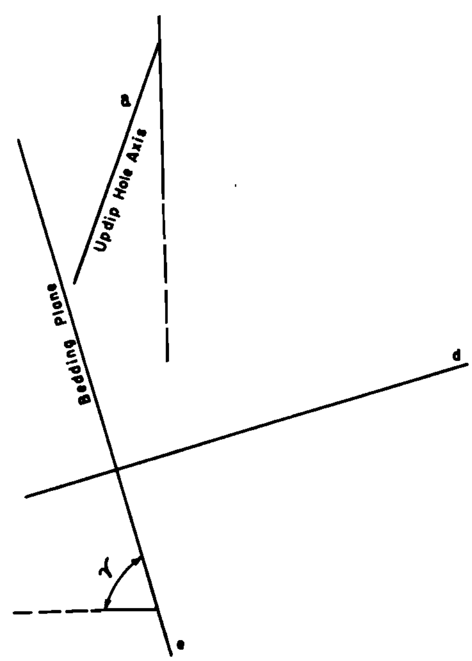

Fig. 21—Nearly Vertical Formations

* See text preceding equation (16).

of nearly vertical formations. It was used for dips of 85 and 80 deg.

Vertical Formations

Applying the above method to the limiting case of vertical formation, formula (19) becomes:

$$\frac{\phi}{\alpha} = 1-h \tag{23}$$

$1-h$ is smaller than unity; and the horizontal line corresponding to such a value of ϕ/α in Fig. 2 intersects some curves at two points, such as H and K. Point K, however, corresponds to an unstable equilibrium and may be disregarded.

The line HK does not intersect the curves of high dimensionless weights, which means that in vertical formations the weight must not exceed a certain value above which there is no equilibrium.* Formula (23) and Fig. 2 were used in preparing Fig. 9i, 10i, and 11i.

DOG-LEGS

Start of a Dog-Leg by a Sudden Change of Weight

Assume that in a straight but inclined hole, a sudden change of weight causes a sudden change of the direction of drilling. The inclination ψ of the instantaneous direction of drilling at that time is given in the general case by formula (17) and for slightly inclined beds by formula (21). The value of $\alpha m/r$, which does not change, is known; and, consequently, the value of ϕ/α corresponding to the new weight may be obtained from Fig. 2 and substituted into (17) or (21).

The sudden change of angle is equal to $\psi-\alpha$. By such a means, curves of Fig. 14 and 15 were computed.

Start of a Dog-Leg by Crossing an Unconformity

Suppose that the beds are inclined above and horizontal below an unconformity. Assume equilibrium conditions before the unconformity is reached and assume that γ is small. Then, making $\psi/\alpha = 1$ in (21), we obtain for conditions above the unconformity:

$$\frac{\phi}{\alpha} = \frac{1-\dfrac{h\gamma}{\alpha}}{1-h}$$

and making $\gamma = 0$ in (21), we obtain for conditions below the unconformity

$$\frac{\psi}{\alpha} = (1-h)\frac{\phi}{\alpha}.$$

* Actually, there is probably an equilibrium for a very inclined hole. This investigation was made with an assumption of small hole inclination.

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† Although this paper as a whole is of value, one portion may mislead the reader, viz., the one concerning a straight-hole drilling technique by subjecting the drilling string to tension. This was shown in the following: 1, Discussion by H. B. Woods of the paper by A. Klinkenberg, "The Neutral Zones in Drill Pipe and Casing and Their Significance in Relation to Buckling and Collapse," *Drilling and Production Practice*, 64 (1951), and 2, Lubinski, Arthur. Influence of Tension and Compression on Tubular Goods in Oil Wells, *Sec. IV, Proc. Am. Pet. Inst. (Prod. Bul. 237)* 31 (1951).

DISCUSSION

J. B. Murdoch, Jr. (Eastman Oil Well Survey Co., Denver, Colo.)(written):* Mr. Lubinski and Mr. Woods are to be commended for presenting the theoretical basis for an analysis of the factors affecting the deviation and dog-legging of drilled wells. They demonstrate mathematically that the drilling of a truly vertical hole to any considerable depth is a practical impossibility. Because of the small amount of weight which it would be necessary to carry on the bit, the penetration rate would be reduced so that the drilling of such a well would be economically unsoun

The authors have advanced a theory that, when drilling in a homogeneous formation with a given drill-collar diameter and hole size and constant weight on the bit, an equilibrium angle will be reached and maintained. This is substantiated by practice. Surveys of many wells in uniform formation where drilling methods are standardized show that the well course assumes and maintains a somewhat uniform angle and direction for most of its length. Variation in the drilling procedure usually changes the drift angle and turns the well course.

One of the proved systems of reducing angle in the drilling of a directional well has been to use a stabilizer above the drill collar and bit, and to drill with reduced weight. Thus, theory bears out practice, because this drilling setup is an effective means of increasing the distance between the bit and the "point of tangency" of the drill pipe.

The authors' demonstration of the influence of the ratio of the drill-collar size to the size of hole cut on the drift angle, is substantiated by another practice developed in drilling directional holes. A drilling setup which increases drift consists of a bit, a stabilizer (or roller reamer) above it, and plain drill pipe used instead of drill collars. Drilling with this setup with increased weight on the bit invariably increases the drift. A variation of this drilling hookup often is used immediately after a deflecting tool has been run. The stabilizer is eliminated and drilling is done with plain drill pipe above the bit. This generally increases drift in the well and continues the directional turn initiated by the whipstock. The theoretical data in this paper shows that the ability of the drill string to bend in the hole tends to increase the "equilibrium angle" of drift. Advantage is taken of this fact to change the course of a directed well.

A study of the deviation of diamond-drill holes might be of interest to investigate this theory further. In drilling such small-diameter holes, rods are used that almost completely fill the hole; in other words, clearance between the drill rod and the walls of the hole is practically zero. In the few surveys I have seen of such drill holes, the deviation has seemed very slight considering the small diameter of the holes and the ability of the rods to bend. Obviously, any study of the subject should take into account the dip of the beds through which the hole is drilled, hole diameter, hole length, the type bit used, the amount of weight applied to the bit, and many other factors.

The effect of clearance between the drill collar and the hole increases only when extreme weights are applied to the bit. The "equilibrium angle" is greatest for extreme weight on the bit. Thus, it would seem best to employ oversize drill collars with weights less than the extreme, and accept a certain moderate amount of drift as a requisite for increased rate of penetration. Instances cited in the paper show that a considerable saving in drilling expense has been realized in allowing the drift angle to increase moderately.

This analysis shows that dog-legs of small magnitude are caused by a sudden increase or decrease in the weight applied on the bit; also that when the bit is drilling in highly inclined beds the dog-leg is much larger. The use of oversize drill collars reduces the change of angle because the clearance between the drill collar and the walls of the hole is less, thus effectively prevents the bit from realigning the well course. The instantaneous change of angle is almost identical, regardless of the amount of weight carried on the bit. Therefore, drilling with increased weight will not result in a sharper dog-leg. The authors prove that the use of oversize drill collars and additional weight on the bit while maintaining a moderate drift angle in the hole will not cause sharper dog-legs in the well course.

An investigation was conducted recently previous to the preparation of a paper dealing with methods used and difficulties encountered in producing directionally drilled wells. Operators who had years of practical experience in producing many wells of this type were interviewed. The consensus was that the inclination of the well (even when drift angles were as great as 75 deg) did not greatly increase production costs. The principal cause of sucker-rod and tubing wear, and much of the pump abrasion, was attributed to dog-legs in the well course. Many of the operators stated that they would prefer to pump highly deviated wells with uniform courses, rather than produce so-called straight wells in which the course of the hole formed a tight spiral, or which contained sharp kinks. Probably a dog-leg limit of 3 to 4 deg per 100 ft of hole drilled would be practical for economical production. Dog-legs in the

*Since removed to Long Beach, Calif.

shallower depths of the well are liable to cause more sucker-rod and tubing wear than those deep in the hole, because more of the weight of moving equipment bears on the side of the hole in the first instance. These production engineers considered that pumping costs could be reduced materially if wells were drilled more carefully in order to eliminate sharp bends in their courses, and that extreme care used to keep drift angles low was unnecessary. From these opinions, it would seem that allowing wells to deviate beyond present straight-hole contract limits would not tend to increase production costs.

Wesley W. Moore (British-American Oil Producing Co., Dallas)(written): The authors have analyzed quantatively the factors affecting the angle of inclination and dog-legging in rotary bore holes. These analyses can serve as a guide for substantial reductions in rotary drilling costs.

I shall confine my comments to the conclusions reached by the authors and, in support thereof, shall present some case histories which I believe are of interest.

Fig. 1 (Moore) represents the drilling progress of the well in Fremont County, Wyoming, referred to by the authors as the one used in my discussion of R. B. McCloy's paper presented in 1952 at Wichita, Kansas. It is reshown at this time for the purpose of illustrating by example the authors' conclusion 2a, viz.: "In crooked-hole territories, cost of drilling may be lowered by carrying much more weight and deliberately accepting a larger deviation; this does not result in sharper dog-legs. Such a method could not be used in steeply dipping formations because unreasonable deviation would result."

It will be noted that, by deliberately accepting the larger deviation, it was possible to reduce rig time an estimated 62 days. Inasmuch as this was a very large rig, it is estimated that the savings on this well approximated $85,000. I do not fully agree with the authors in their statement that "such a

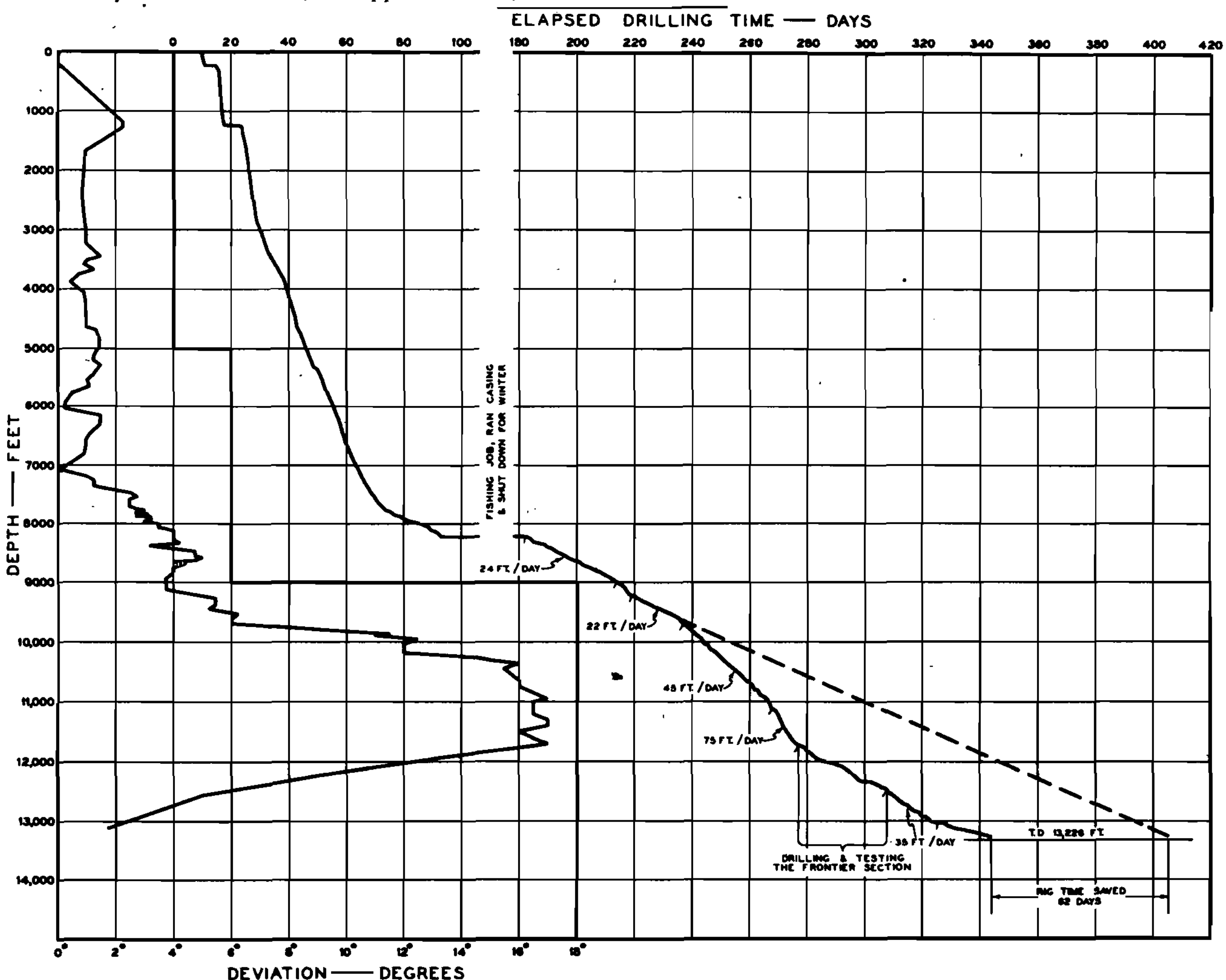

Fig. 1 (Moore)

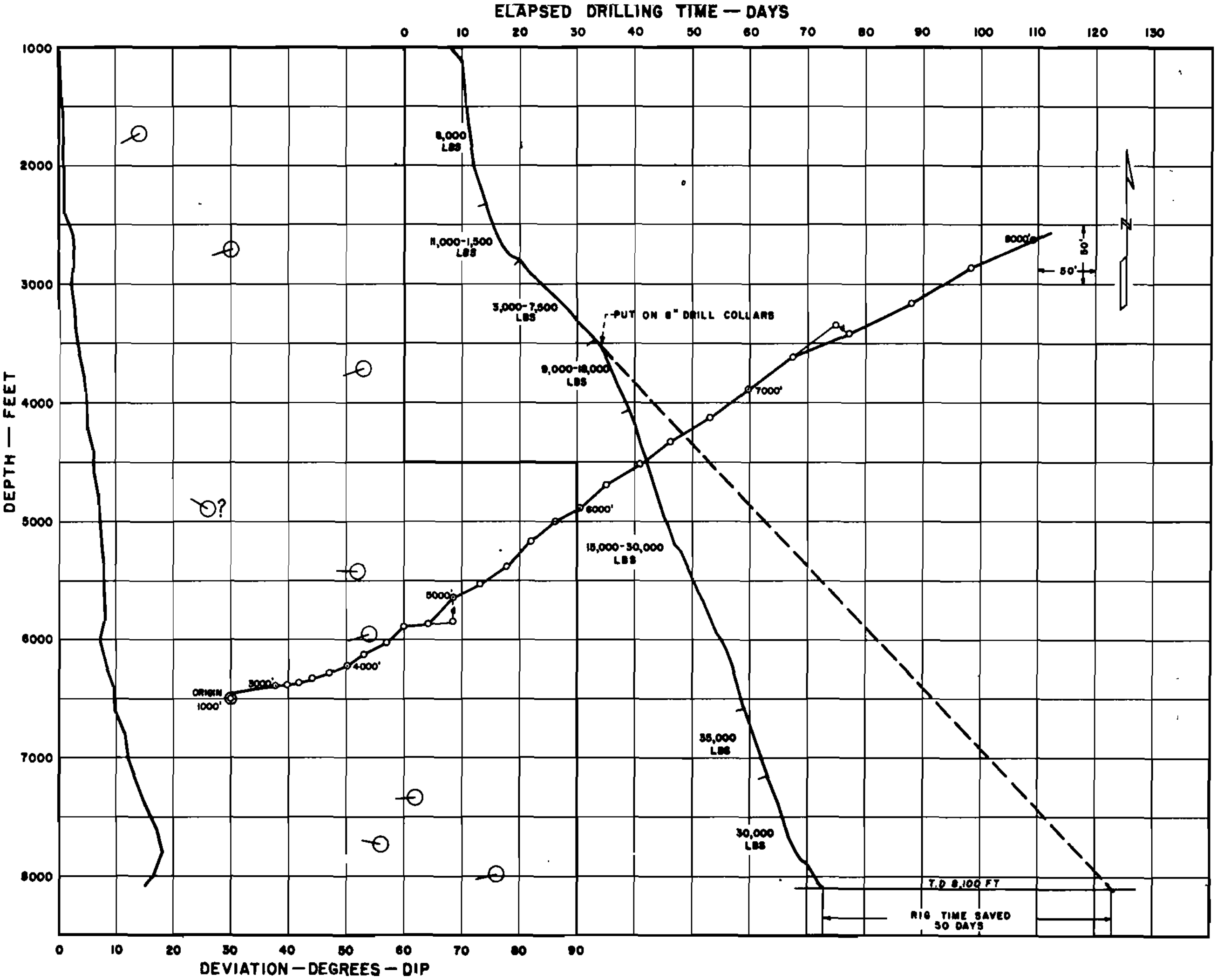

Fig. 2 (Moore)

method cannot be used in steeply dipping formations because unreasonable deviation would result."

Fig. 2 (Moore) represents the drilling progress of a well in which very steeply dipping beds were encountered and which was drilled rapidly without creating serious dog-legs, even though the angle of deviation climbed to as high as 18 deg. This particular well was a wildcat drilled on a lease across an overthrust fault from the main part of the field, and the surface location was so situated as to permit a drift even larger than was experienced without either crossing the fault or going beyond the lease boundary. The drilling of this well, because of lease commitment, was an operation against time. Therefore, it had to be drilled rapidly at whatever cost.

Difficulties experienced in maintaining straight hole from the surface down to about 3,500 ft made it appear improbable that we could successfully drill to desired depth in the allotted time using 6¼-in. drill collars. Therefore, 8-in. drill collars were used in a 9-in. hole below 3,524 ft. Weights carried are noted along the drilling progress curve. Dog-legs can be noted at 4,800 ft and at 7,200 ft, but no drilling difficulty was experienced in the entire 8,100 ft of hole.

The cost of drilling this dry hole was comparable to the drilling of other wells in the field where dips of the formation do not exceed 12 deg. It is difficult to estimate what the cost would have been had we refused to accept so large a deviation from vertical.

In support of the authors' conclusion 2b, viz., "using larger drill collars with which more weight may be carried without increasing deviation. Examples: 8-in. collars in 9-in. hole, 11-in. collars in 12¼-in. hole," I offer Fig. 3 (Moore) which shows the drilling progress at the Steamboat Butte Field,

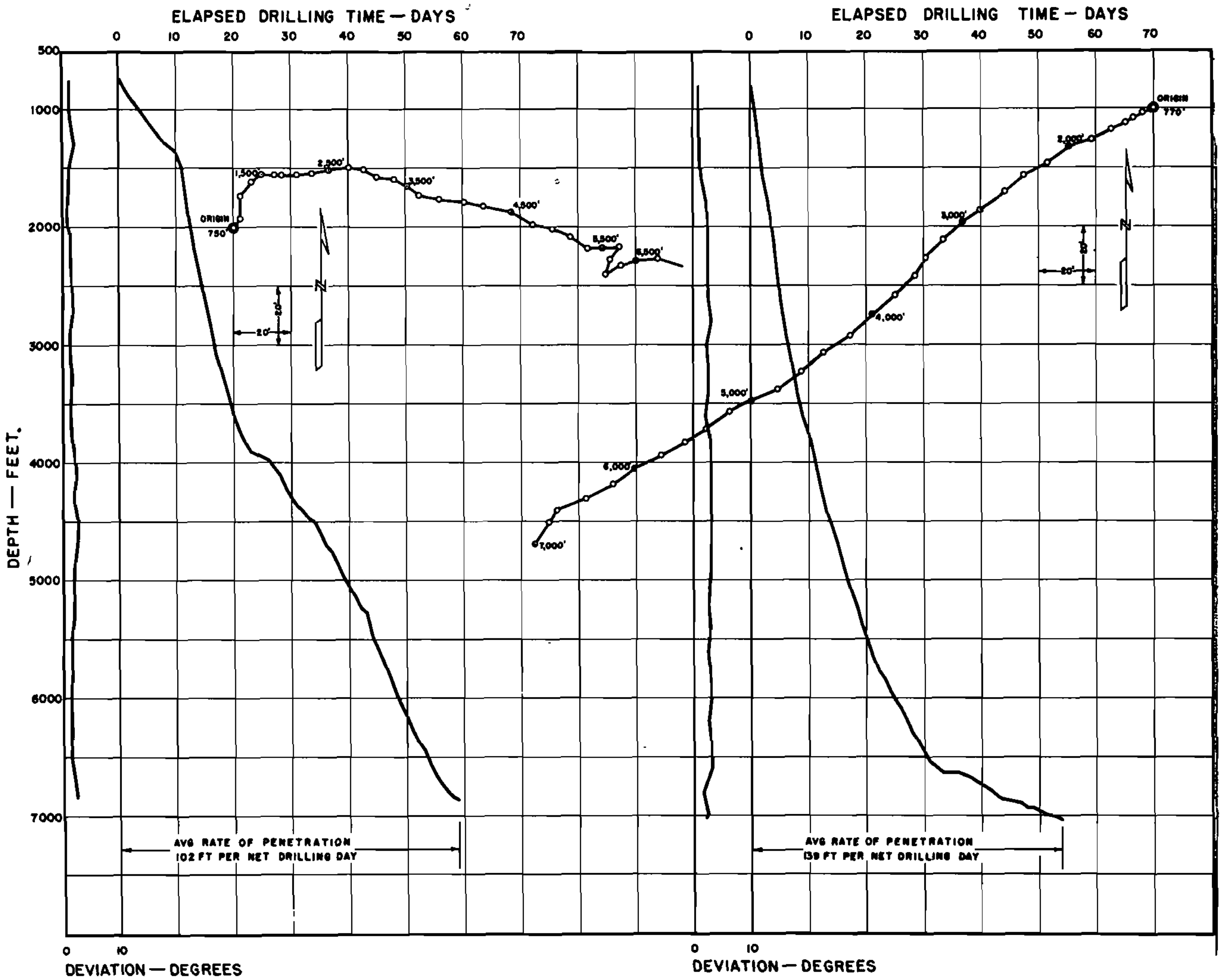

Fig. 3 (Moore)

Wyoming, of two wells drilled with the same rig, using the same crews and pusher. Both wells were located in areas of relatively flat dip. One, however, was west of the axis while the other was east of the axis. The principal difference in the two wells was the use of 8-in. drill collars in a 9-in. hole at the well on the right; whereas 6¼-in. drill collars in a 9-in. hole were used at the well on the left.

Weight on bit for the well on the right averaged something like 30,000 lb with a maximum of 45,000 lb; whereas the weight on bit for the well on the left averaged about 20,000 lb with a maximum of 35,000 lb. It can be noted that the maximum deviation for the well on the right was 3 deg as compared to 2¾ deg for the well on the left.

Average rate of penetration per net drilling day for the well on the right was 139 ft, as compared to 102 ft for the well on the left. Because the price per foot for drilling in this field is based on the average rate of penetration per net drilling day, a savings of $2.71 per foot resulted from the use of 8-in. drill collars; and for a 7,000-ft well, this savings amounted to very nearly $19,000.

Similar comparison can be made between two other similar wells drilled with a smaller rig in this same field, one of which used 6¼-in. drill collars, and the other 8-in. drill collars. The average rate of penetration using 6¼-in. drill collars in a 9-in. hole was 80 ft per day, as compared to 104 ft per day for the same rig, using 8-in. drill collars in a 9-in. hole. Here again, the maximum deviation was slightly greater for the well making the better time, viz., 5¼ deg as compared to the 4¼ deg for the well making the slower speed, but neither hole has given trouble.

The savings in footage cost amounted to $2.63

per foot for the well using 8-in. drill collars, or over \$18,000 for a 7,000-ft well.

Peter A. Szego (The Rice Institute, Houston)(written): The authors are to be congratulated on their valuable contribution to the theory of bore holes.

The mathematical techniques used appear to be correct. An independent check of the numerical computation would be an extensive task—testimony to the magnitude of the work carried out by the authors. However, it is interesting to note that verification of a key numerical value is available in a German paper by A. Willers.* Willers considers only vertical holes ($\alpha = 0$). However, as the authors point out, the functions which must be computed in the inclined case coincide with those required for the vertical case. Willers finds that the weight on bit (in dimensionless units) required to produce first order buckling of a uniform drill stem of infinite length is 1.88. This agrees exactly with the value found by A. Lubinski,† which is the foundation for the present work. Thus there is good reason to trust the accuracy of the calculations.

The agreement is all the more striking when it is realized that Willers employs asymptotic series for large arguments, whereas Lubinski uses power series throughout. Inasmuch as the latter converge slowly for large values of the argument, asymptotic series should be used in the event further investigations require precise calculations in this range.

Actually it will be simpler to carry out further work by approximate methods such as the authors' iteration scheme. In the use of approximate methods the choice of techniques is partly a matter of personal preference. A particularly good feature of the authors' approach lies in the fact that only integrals of the approximating function appear in the final expressions. This means that, even though the approximation is poor in some regions, the average obtained by integration still gives good values. The accuracy of the method is shown by the agreement obtained with values computed from the series solution. Of course, one could carry out a further iteration, but only at the price of greatly complicating the formulae.

The most frequently occuring assumption in this paper is that the angle of inclination of the bore hole be small. The authors estimate that the consequent errors are negligible for angles less than 10 deg; small for 15 deg; and considerable above 20 deg. Because the errors arise primarily through replacing cosines of small angles by one and tangents of small angles by the angles, the order of magnitude of the overall error is obtained by checking the errors arising through these replacements. For angles less than 10 deg, the maximum error for these replacements is about 1½ percent; for angles up to 15 deg, about 3½ percent; for angles up to 20 deg, about 6 percent. Thus the authors' estimates seem reasonable.

The authors point out the desirability of extending their work to large angles of inclination. Inasmuch as the elementary beam formulae are restricted to situations wherein the slope of the deflection curve remains small, the large-angle situation must be studied with inclined coordinates such as those shown in Fig. 1 (Szego). This figure corresponds to Fig. 19 of the paper. L is the distance from the bit to the point of tangency; U and W are the force components acting on the bit; the distributed weight p has a component $p \cos\alpha$ in the X direction and a component $p \sin\alpha$ in the Y direction. The bending equation becomes:

$$EI\frac{d^2Y}{dX^2} = UX - WY + \int_0^X [(Y-\eta)p\cos\alpha + (X-\xi)p\sin\alpha]\,d\xi$$

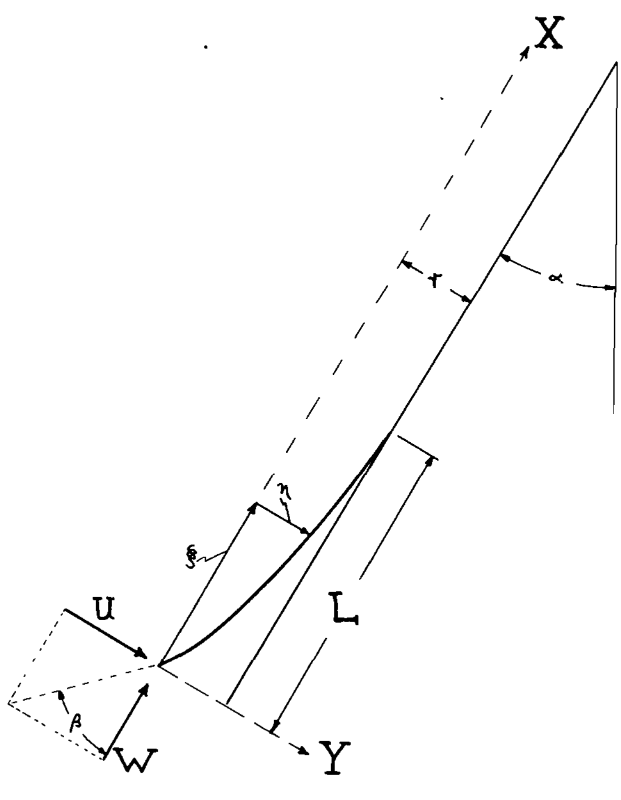

Fig. 1 (Szego)

* Fr. A. Willers: The Buckling of Heavy Rods (in German), *Zeitschrift für Angewandte Mathematik und Mechanik*, **21**, 43 (1941).

† Ref. 1, p. 242.

This generalizes equation (8) of the paper and reduces to it when $\alpha = 0$. Introducing the dimensionless quantities $x = X/m, y = Y/m, l = L/m, u = U/mp, w = W/mp$, the bending equation becomes:

$$\frac{d^2y}{dx^2} = ux - wy + \int_0^x [(y-\eta)\cos\alpha + (x-\xi)\sin\alpha]\, d\xi$$

The boundary conditions in dimensionless units are $y = y''(x) = 0$ at $x = 0$; $y = r/m$, $y'(x) = y''(x) = 0$ at $x = l$.

As a first attempt at a solution, one might try the iteration technique with an initial approximation function:

$$y = \frac{r}{m}\left[\frac{x}{l} + \frac{1}{\pi}\sin\frac{\pi x}{l}\right]$$

Equations (13), (14), and (15) of the paper are then replaced by the respective formulae:

$$\frac{\tan\beta + \tan\alpha}{\tan\alpha} = \frac{r}{m\tan\alpha}\left[\frac{1}{l} - \frac{\cos\alpha}{w}\left(\frac{1}{2} - \frac{2}{\pi^2}\right)\right] + 1 - \cos\alpha\,\frac{l}{2w}$$

$$\frac{\tan\beta + \tan\alpha}{\tan\alpha} = \frac{r}{m\tan\alpha}\left[\frac{1}{l}\left(1 + \frac{3}{\pi^2}\right) + \frac{\cos\alpha}{w}\left(\frac{18}{\pi^4} - \frac{3}{2\pi^2} - \frac{3}{8}\right) - \frac{3}{wl^3}\right] + 1 - \frac{3}{8}\cos\alpha\,\frac{l}{w}$$

$$\frac{r}{m\tan\alpha} = l\cos\alpha \div \left[\frac{24}{l^3} - \frac{24}{\pi^2}\frac{w}{l} + \cos\alpha\left(\frac{28}{\pi^2} - \frac{144}{\pi^4} - 1\right)\right]$$

When α and β are small, these expressions reduce to those obtained by the authors. One need only note that $\alpha + \beta = \phi$.

Using the foregoing expressions one can compute corrections for the curves developed by the authors and extend their range to larger angles. Checks should be made to determine the range of values of α for which the approximation is sufficiently accurate. The case $\alpha = 90$ *deg* can be studied directly with fairly simple functions. It is to be hoped that the large-angle case will be investigated further.

Mr. Lubinski and Mr. Woods (written): The authors wish to thank Dr. Szego for his comments. They particularly appreciate his outline of a procedure for eliminating the simplifying assumption of small-hole inclination which undoubtedly will be useful in future research.

The authors wish to thank Mr. Moore for his valuable discussion. His actual field examples confirm the conclusions of the paper, based on theoretical findings. He does not wholly agree, however, with the conclusion that in steeply dipping formations, carrying heavy weight would result in unreasonable deviations. Mr. Moore has perhaps misunderstood our conclusion 2a, which for additional clarity should have been worded as follows (new text is italicized:

2. In crooked-hole territories, cost of drilling may be lowered by:
   a. *Using conventional drill collars, but* carrying much more weight *than generally done* and deliberately accepting a larger deviation; etc.

In Mr. Moore's example of his Fig. 2, 8-in. OD, i.e., oversize, drill collars are used below 3,500 ft and this chart is a good example for our conclusion 2c rather than 2a.

The authors wish to thank Mr. Murdoch for his interesting comments. The most important of these may be summed up as follows:

1. Production troubles are caused by dog-legs.
2. Inclination of the well (even with drift angles as great as 75 deg) has very little effect on production difficulties.
3. The paper might be used to evaluate the performance of slim-hole drilling.

The second comment is very significant in view of the fact that the authors recommend drilling holes inclined up to 15 or 20 deg only.

The authors are in agreement with the third comment above. Cheap slim-hole drilling, recently advocated by Gerald Burgess, may become in the future a common practice in drilling exploratory wells. One of its limitations is that the method is very likely not suitable in very dipping formations. This investigation could make possible a comparison of the relative merits of slim-hole and conventional-hole drilling for any given set of conditions. For this purpose, curves for greater dimensionless weights than those made so far should be prepared.

J. P. Bernhard (Regie Autonome Des Petroles, Paris, France)(written):* This paper throws a very new and bright light on the whole problem of wellbore drift. After thorough study and comparison with experimental facts, I cannot find anything against the statements presented.

I should like, however, to add the following comments to this very valuable work.

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* Prepared following presentation of paper.

Orientation of the Drift

The authors start from the well-known fact that the bit generally drills updip and, by introducing the conception of a drilling anisotropy index, arrive at the calculation of the equilibrium angle.

I wish to point out the consideration of the anisotropy index is sufficient to explain that the bit can generally not reach any equilibrium condition out of a vertical plane perpendicular to the bedding plane.

Let us choose, as axes of coordinates, O_x and O_y in a horizontal plane, O_y being a horizontal of the bedding plane, and O_z an upward oriented vertical.

As auxiliary axes, we choose O_y' coincident with O_y, O_z' normal to the bedding planes, and O_x' in the bedding plane, oriented downdip.

Let ϕ and θ be, respectively, the inclination and the orientation of the force on bit, whose magnitude we take as unity.

The components of the force on bit on the axes O_x, O_y, and O_z are:

$$\begin{cases} X = \sin\phi \cos\theta \\ Y = \sin\phi \sin\theta \\ Z = \cos\phi \end{cases}$$

Projecting on the axes O_x', O_y', O_z', we find the components:

$$\begin{cases} X' = \sin\phi \cos\theta \cos\gamma - \cos\phi \sin\gamma \\ Y' = \sin\phi \sin\theta \\ Z' = \sin\phi \cos\theta \sin\gamma + \cos\phi \cos\gamma \end{cases}$$

The components of an instantaneous displacement with respect to the axes O_x', O_y', O_z', will be proportional to:

$$\begin{cases} V_x' = (1-h)\, X' \\ V_y' = (1-h)\, Y' \\ V_z' = Z' \end{cases}$$

and with respect to the axes O_x, O_y, O_z, to:

$$\begin{cases} V_x = (1-h) \cos\gamma\, X' + \sin\gamma\, Z' \\ V_y = V_y' \\ V_z = -(1-h) \sin\gamma\, X' + \cos\gamma\, Z' \end{cases}$$

Substituting, and rearranging, we obtain:

$$\begin{cases} V_x = \sin\phi \cos\theta \\ \quad -h \cos\gamma\, (\sin\phi \cos\theta \cos\gamma - \cos\phi \sin\gamma) \\ V_y = (1-h) \sin\phi \sin\theta \\ V_z = \cos\phi - h \sin\gamma (\cos\phi \sin\gamma - \sin\phi \cos\theta \cos\gamma) \end{cases}$$

The orientation η of the displacement will be given by:

$$\tan\eta = \frac{V_y}{V_x} = \frac{(1-h) \sin\phi \sin\theta}{\sin\phi \cos\theta - h \cos\gamma (\sin\phi \cos\theta \cos\gamma - \cos\phi \sin\gamma)}$$

Writing $\eta = \theta$ (equilibrium condition) we obtain, after rearranging:

$$h \cos\phi \cos\gamma \sin\gamma \sin\theta \left[\tan\phi \tan\gamma + \frac{1}{\cos\theta} \right] = 0$$

Besides the solution $h = 0$ (isotropic formation) and $\gamma = 0$ (horizontal formations), which were obvious, this equation is satisfied by $\theta = 0$ (downdip drift) and $\theta = \pi$ (updip drift).

Still another equilibrium condition would be realized by satisfying the equation:

$$\cos\theta \tan\phi \tan\gamma + 1 = 0$$

For drifts which are not abnormally high, say about 20 deg, the solution of this equation has a physical meaning only if γ is greater than 70 deg.

Thus we may conclude that whatever the drift angle and the formation should be, a stable condition can be found for updip or downdip drift only, unless a definite relationship exists between dip, drift angle, weight on bit, and direction of drift. This relationship can be maintained only in extreme cases of nearly vertical dip or extreme drift.

However, cases of lateral drift are known even in current practice. The above statement means that their interpretation needs something else than the anisotropy index, such as rotational or frictional effects, etc.

Hole Inclination in Anisotropic Formations

I would suggest a simpler way to arrive at the calculation of the drift angle. Consider Fig. 1 (Bernhard) the projections of the force on the bit on the axes d and e used by the authors. They are (the magnitude of the force on bit being again taken as unity):

on axis d: $\cos(\phi-\gamma)$

on axis e: $\sin(\phi-\gamma)$

The components of the instantaneous displacement are proportional to $\cos(\phi-\gamma)$ and to $(1-h)\sin(\phi-\gamma)$

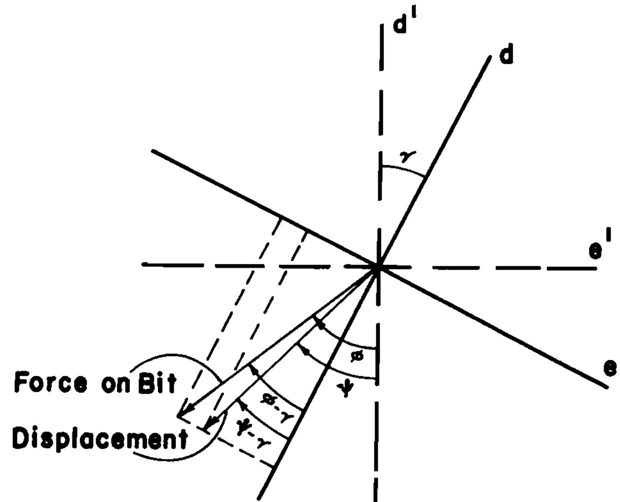

Fig. 1 (Bernhard)

So one can write immediately:

$$\tan(\psi - \gamma) = (1 - h)\tan(\phi - \gamma)$$

which is valid for any angle. For small angles, it is easy to show that this equation is the same as equation (17).

For equilibrium conditions, we write:

$$\tan(\alpha - \gamma) = (1 - h)\tan(\phi - \gamma)$$

This equation can be used in a very easy way: γ being given, to each arbitrary value of α corresponds a value of ϕ, so ϕ/α may be calculated. Then, for a given weight on bit, corresponding values of $\alpha(m/r)$ are found on Fig. 2 of the authors paper. Dividing α by those values, the values of r/m are obtained.

Dog-Legs

I agree with the authors' conclusion that the dog-legs caused by unconformities are not sharper in inclined than in nearly vertical holes. However, this holds true only insofar as the instantaneous change of angle is concerned. The equilibrium angle on the contrary is much more sensitive to changes of dip in inclined holes. Referring, for instance, to Fig. 13, one can note that the equilibrium angle varies from 18 deg to 25 deg on curve 8 if the dip varies from 25 deg to 35 deg; whereas it varies only from 2.2 deg to 3 deg on curve 1. It is my belief that this could lead to some dog-legging, if very inclined holes are drilled with a large clearance.

We had recently an example of such changes in our operations in Southern France. A well had been sidetracked in an updip direction. The plans were for a 20-deg inclination which has been obtained very easily, the average dip being 30 deg. However, the formation drilled is well known for having local variations in dip of about ±5 deg. These variations caused the inclination to vary rather quickly between 16 deg and 22 deg, without any change in weight on bit. No trouble has been experienced during drilling, however.