Abstract
A tapered electric submersible pump (ESP) is mainly used to pump wells with high gas oil ratio. Free gas is separated and vented via a shroud or gas separator. Or, it is compressed using a tapered larger-than-normal pump or specially-designed gas handler below the “normal” pump. Although tapered ESP has been used for decades in petroleum production, few articles have discussed its design. After studying the pressures and flow rates stage by stage using a computer program, the paper presents basic criterion to design a tapered pump.

Free gas in pumped fluid stream reduces pump performance, and may cause surging and gas lock. For tapered pumps the free gas effect becomes vital since generally tapered pumps handle considerable amount of free gas. The paper discusses traditional homogeneous model and multiphase pumping model. By comparing pump performances of the two models using examples, the paper presents that the traditional model designs fewer stages and will produce smaller rate than desired rate. Further, without considering free gas effect, the pump above bottom pump may work out of its operating range. For a tapered pump, pumping stability should be checked and pump degradation should be included in stage by stage calculation. Also, fluid flow pattern should be checked to avoid slug flow at the place of pump intake.

Also presented are optimal design methods for both single and tapered pumps. Widely used design methods are using desired liquid rate at surface or the liquid rate at pump intake to select a pump with closest best efficiency point. The paper illustrates by examples that the two liquid rate methods fail to design high efficiency when pumping high gas/liquid fluid, and proposes two methods of using total rate at pump discharge and using average total rate. The two design methods will improve a well’s pumping efficiency and running life.

Introduction
Electric submersible pump (ESP) is a widely used artificial lift method especially for producing large volume fluid. The base to design an ESP is its standard pump curve (or called standard catalog graph) provided by ESP manufacturers. As shown in Fig. 1, the standard pump curve generally consists of three performance curves, pump head performance, brake horsepower performance and efficiency performance. Also, marked on the graph is the pump’s operating range. The pump should be operated in the recommended range to avoid low efficiency and mechanical damage. The performances are measured at laboratory condition with pure water at standard frequency (60 Hz or 50 Hz).

Another important parameter for selecting an ESP is its best efficiency point (BEP) as marked in Fig. 1. It is sometimes called design point and, in most cases, it is the rate at which the efficiency is the highest. Therefore, a widely used method in an ESP design is to find an ESP with a BEP closest to desired production liquid rate at pump intake condition. The BEP divides the operating range into lower and upper ranges.

The low end of the operating range is generally set by maximum downward thrust force with reasonable pump life, and the high end is set by a value which is a little bit less than neutral thrust point. When a pump works beyond its high end, the pump may generate an upward thrust force that is harmful to pump life. On the other hand, once pumping fluid goes below the pump’s low end, the thrust bearing of the pump may suffer excessive wearing.

Although a pump’s standard catalog graph is a major reference for pump design, one may not use it directly in oil wells. Pump manufacturers generate the graph at standard temperature and frequency (60 Hz or 50 Hz) with pure water. At wellbore condition, fluid may not be pure water and its viscosity may affect the pump’s performance. When pumping viscous fluid, pump brake horsepower will increase, and its capacity, head and efficiency will decrease. Therefore, the standard catalog graph should be corrected for viscosity before used in well condition. One may also correct viscous fluid to equivalent pure water and use the standard catalog graph. The correcting correlations for viscosity are
\[ q_{\text{vis}} = C_q q_w \]
\[ H_{\text{vis}} = C_H H_w \]
\[ E_{\text{vis}} = C_E E_w \]
\[ HP_{\text{vis}} = C_{HP} HP_w \]

Where \( Q, H, E, \) and \( HP \) are fluid rate, head, efficiency, and brake horsepower. The correcting factors \( C_q, C_H, C_E \) and \( C_{HP} \) are determined by fluid viscosity and can be read from graphs or tables given by Brown (1980). Fig. 2 shows an example of correction form pure water to viscous fluid. It is the corrected standard catalog graph for the pump in Fig. 1 when pumping fluid has a viscosity of 700 SSU (Saybolt Seconds Universal).

If the specific gravity of the viscous fluid is different from that of pure water, pump horsepower should be corrected for gravity by multiplying the specific gravity of the viscous fluid. Well fluid viscosity varies with pressure, temperature, water cut and gas/liquid ratio. During production, those variables vary with producing rates and pump stages. The viscosity correction should be made stage by stage.

In addition to correcting viscosity and gravity of pumping fluid, the standard catalog graph should also be corrected for pump rotation speed if it is different from standard frequency. The variation of pump performance with its rotation speed follows the affinity laws. According to the laws, fluid flow rate varies proportionally to the speed. Pump head is proportional to the square of the speed, while pump brake horsepower is proportional to the cube of the speed. Only pump efficiency remains constant with the speed variation.

\[ \frac{q_2}{q_1} = \frac{N_2}{N_1} \]
\[ \frac{H_2}{H_1} = \left( \frac{N_2}{N_1} \right)^2 \]
\[ \frac{HP_2}{HP_1} = \left( \frac{N_2}{N_1} \right)^3 \]

In the following, pump standard catalog graph is corrected to in-situ condition. Pump capacity and the performance curves of head, efficiency and brake horsepower are all corrected for viscosity, specific gravity and frequency. So do the high and low ends of operating range and the BEP of a pump. For convenience, we may not mention the word “corrected” in follows.

Gas usually accomplishes with oil production. The gas may come from gas cap in reservoir or be released from solution gas in oil as pressure decreases. During production, fluid pressure decreases from reservoir pressure to flowing bottomhole pressure, and will further reduces to the pressure at pump intake place if the pump is set above perforation. Due to the high compressibility property of gas, the fluid of gas/liquid mixture, which is in a pump’s operation range at pump intake place, may be beyond the range at pump discharge place. Or the fluid satisfies the operation range at pump discharge but may not fall in the range at pump intake.

For a highly compressible fluid (gas/liquid mixture), to make its production rate in a pump’s operation range, a tapered pump is used (Swetnam and Sachash, 1978). Tapered pump is a series of two or more pumps with the largest volumetric capacity pump at bottom and the smallest pump at top. The largest capacity pump is mainly used to compress and mix gas/liquid mixture and smallest pump works mainly for generating head. The combination gives a tapered pump a wide operating range. Generally, those pumps in a tapered pump have the same outside diameters and are not tapered from their outside sizes and shapes. As shown by Swetnam and Sachash (1978), the design of a tapered pump is to make production fluid rate in the operating ranges of all stages from pump intake to discharge.

The presence of free gas in pumping fluid will degrade pump performance, and may even bring surging or gas lock as gas void ratio gets some values. Lea and Bearden (1982) studied the effects of gas on pump performance experimentally. They concluded that gas degradation on pump performances depends upon pump type (radial or mixed flow path), free gas volumetric percentage (void ratio) and pressure at pump intake. The lower the pressure and/or the higher the void ratio is, the severer the degradation. At high pump intake pressure, pump performance will not be affected much by free gas.

Many authors have studied the effect of free gas on pump performance. Sachdeva et al. (1988) studied the achievements in multiphase pumping by centrifugal pumps in nuclear industry, and developed a dynamic model to simulate ESP performance in multiphase condition. The theoretical model is too complex to use for most petroleum engineers, Sachdeva et al. (1992) presented a simplified correlation to model the two phase pumping performance. Most recently, Beltur et al. (2003) summarized a series of experimental studies and analyzed the gas effects from those data.
In field application, free gas is generally separated to avoid being ingested by a pump and vented through the annular of tubing and casing by using a shroud or a gas separator. Free gas is harmful for pump performance but it is helpful in flowing fluid to surface like the effect of gas lift.

In a tapered pump, the bottom pump is usually a regular pump with fewer stages. It may also be a specially designed pump, called gas handler by some manufacturers. Gas handler could pump higher gas fraction without surging and gas lock. In recent years, especially with the development of specially designed pump to handle gas, tapered pump become popular.

Although tapered pump has been used in petroleum industry for decades, lower-than-estimated efficiency is still common. Very few papers discussed the performance and design of a tapered pump. This paper discusses traditional homogeneous model and its limitation to handle gas, the application of performance degradation model, tapered pump design, as well as optimal design methods to improve pumping efficiency for both single and tapered pumps in gassy wells.

Pump Design

Design Principle. The design of an ESP is to select a proper pump and calculate required pump stages for a desired liquid rate (DLR). The pressures at the places of pump intake and discharge are needed to obtain in-situ flow rates and viscosities for pump capacity checking, and to calculate required total dynamic head for stage calculation. DLR refers generally the liquid rate at surface condition. Using the DLR, water cut, gas oil ratio (GOR), fluid properties and well configuration, one may calculate the in-situ pressures, viscosities, and fluid rates.

Using reservoir inflow performance correlation, one may easily obtain flowing bottomhole pressure for the DLR. Pump intake pressure can be calculated from the flowing bottomhole pressure to the place of pump intake by multiphase flow correlation for the DLR and fluid properties. Downward from well head, the pressure at a pump’s discharge can be calculated.

At pump intake, some gas may be separated and vented through tubing/casing annular, the gas/liquid ratio and fluid bubble point pressure of the fluid stream passing through a pump need recalculated. The above calculations can be done by most commercial software. Fig. 3 shows a calculated pressure profile for a well in Table 1 without gas separation. The pump will be set at a measured depth of 7000 ft. Pump length is ignored here.

The differential pressure at the places of pump discharge and intake is the pressure a pump should generate to flow the fluid. The stage number of the pump is calculated from the discharge pressure and intake pressure. The differential pressure can be changed to total dynamic head (TDH). The stages, \( n \), of a pump needed to generate the head is

\[
n = \frac{TDH}{H}.
\]

Eq. 3 assumes that each stage generates the same head, \( H \), and fluid density is the same for all stages. In fact, both the density and each stage’s head vary with stage number.

As pressure increases in pump stages, free gas may be compressed back into solution or just compressed. Fluid volume reduction for gas back-into solution and compression are different as shown in Fig. 4. The symbol ‘O’ marks the average rate from pump intake to discharge in Fig. 4. For the Low GOR curves, gas void ratios are 5.7% and 5.77% at pump intake for compression and back-into solution respectively. For the High GOR curves, gas void ratios are 49.63% and 49.43% for compression and solution. The stage number is counted from pump intake to discharge.

Gas will release from oil as pressure decreases and should go back into oil as pressure increases. Fluid rate reduction depends on gas void ratio. However, since pump rotation speed and fluid velocity in an ESP are very fast, the free gas may don’t have enough time to solute into oil. The actual situation may be between back-into solution and compression, and more close to compression around standard rotation speed.

The calculated flow rates at pump intake and discharge places are used to check against a pump’s operating range. The rate at pump intake should be lower than the high end and the rate at pump discharge should be higher than the low end of the operating range of a selected pump. Out of the operation range, in addition to excessive wearing, the pump doesn’t work efficiently.

If none available pump satisfies the rates at pump intake and discharge places, one may turn to a tapered pump. For the design of a regular single pump, as shown in Fig. 3, one can calculate the pressures at pump intake and discharge places for a DLR. Thus obtains the fluid rates at the two places before selecting a pump. However, the design of a tapered pump needs lower pump information to design an upper pump.

Before selecting pumps (bottom, top and/or middle pump) for a tapered pump, one can only know the pressures and rates at the places of the intake and discharge of the tapered pump, which is the bottom pump intake and the top pump discharge. One needs to know the pressures and rates at each pump’s intake and discharge to select a pump and calculate its stages. Therefore, the procedure of design a tapered pump is trial and error; generally starts form the bottom pump.

More detail, use the calculated fluid rate at the place of the tapered pump intake to select a bottom pump whose high operating end is greater than the calculated fluid rate. And assume a stage number for the selected bottom pump, and calculate the selected pump’s discharge pressure and flow rate. Then use the discharge pressure and rate of the bottom pump
as the intake pressure and rate for an upper pump. If the upper pump is top pump, calculate its required stages using the
known discharge pressure and rate and the calculated intake pressure and rate. Otherwise, assume its stages and follow the
above procedure until top pump. Last adjust the assumed stages and repeat the procedure until the design is satisfied.

Unlike single pump design, a tapered pump design follows trial and error method by adjusting assumed pump stages. Eq.
3 can only estimate stage number due to the difference of the flow rate and pump head in each stage. For both single and
tapered pump designs, flow rate, head and efficiency should be calculated stage by stage.

**Homogeneous Model.** In addition to the correction of viscosity and frequency on standard catalog graph, free gas in
pumping fluid also affects pump performance and should be corrected. Traditionally, the gas effect is ignored by assuming
that gas/liquid mixture behaves like a single liquid. The assumption is called homogeneous model and it is correct if the free
gas is in tiny bubbles and the bubbles are mixed homogeneously with liquid phase. Practically, homogeneous model is
acceptable if the gas void ratio at pump intake is less than 10%.

For homogeneous model, one may directly use pump performance curves corrected for viscosity and frequency, like Fig.
2, by using the in-situ total rate of water, oil and gas in a gas/liquid mixture as the rate on the rate dimension. One may also
change the performance curves by keeping the rate dimension as liquid rate. To use the changed pump curves, only the in-situ
liquid rate (water rate and oil rate) in a gas/liquid mixture should be used. It should be noticed that the in-situ total rate or
liquid rate is a corrected rate for the mixture viscosity of total fluid (water, oil and gas) or liquid (water and oil).

When using liquid rate on rate dimension, pump performances vary with gas void ratio. Fig. 5 gives the head performance
curves from homogenous model. In Fig. 5, dimensionless head and dimensionless liquid rate are used. They are the ratios of
liquid rates and corresponding heads to the pump’s maximum rate and maximum head respectively.

For the well in Table 1, the total rates at the places of pump intake and discharge are 1271 B/D and 727 B/D. No pump
has an operating range covering the rates from a manufacturer. Pump B in Fig. 6 is the closest pump whose high operating
dend covers the rate at pump intake (1271 B/D). Its corrected operating range is 861 to 1625 B/D with 1226 B/D of BEP. If
select Pump B for the well, from commercial ESP software the calculated stages are 196 for compression gas situation.

Among the 196 stages, 129 stages (from stage 67 to stage 196) work below the low end of the pump (861 B/D). Obviously,
the pump would suffer excessive wearing if the pump were used for the well.

To design a tapered pump is to select pumps to make the tapered pump’s operating range cover the total fluid rates at the
places of pump intake and discharge. Select a large pump as bottom pump whose high operating end just above the total rate
at pump intake. Then select a smaller upper pump whose operating range overlaps with the bottom pump. Last the low end of
a top pump should be just below the total fluid rate at pump discharge.

For example, use the Pump B in Fig. 6 as a bottom pump, its high operating end (1625 B/D) is larger than the pump
intake rate of 1271 B/D. Select Pump A in Fig. 1 as Pump B’s upper pump, the operating ranges of pumps A and B overlap.
The low operating end of Pump B (861 B/D) is lower than the high operating end of Pump A (899 B/D)). Pump A is also the
top pump of the tapered pump with a BEP of 671 B/D, and the low operating end of pump A (378 B/D) is lower than the total
fluid rate at pump discharge (727 B/D).

Design from bottom to top pump, using trial and error method, Pump B needs 50 stages to compress the fluid rate from
1271 B/D to 896 B/D which is in the operating range of Pump A (378-899 B/D). The discharge pressure at stage 50 is 460
psia. Calculating the stages for pump A using the intake pressure of 460 psia and the discharge pressure of 1659 psia gets 172
stages.

It should be noted that the exact calculated stages are in 171 to 172. The actual stage number is rounded up. Therefore, the
production liquid rate (608 STB/D) and the pressure profile of the tapered pump are a little bit different from those of using a
single Pump B.

**Gas Interference Model.** Since free gas will degrade pump head performance, using homogeneous model without
correction for gas effect will overestimate head gain of a pump in gassy wells, and hence overestimates liquid compression
and volume reduction and gives fewer stages.

Therefore, using homogeneous model, a pump’s top stages may work at rates below the pump’s low operating end for a
single pump design if the rate at the pump’s discharge place is designed close to the low end. For a tapered pump, the rates at
an upper pump’s intake may be larger than the pump’s high end.

Homogenous model assumes that gas bubbles are very small and mix homogeneously with liquid phase. It can be used if
the pressure is high and/or gas void ratio is low. For a low pressure and/or high gas void ratio case, pump head degrades a lot,
and correction for gas should be considered.

A few models were presented as summarized by Beltur et al. (2003). In the paper, we use the model presented by Zhou
and Sachdeva (2006) as,

\[
\frac{H_m}{H_{max}} = K(p_m)^{\alpha E_1}(1 - \alpha)^{E_2}\left(1 - \frac{q_m}{q_{max}}\right)^{E_3}.
\]  

(4)
Where $H_m$, ft, is the head gain from a pump stage, $H_{max}$, ft, is the maximum head at zero flow rate of the stage, $p_{in}$, psi, is the pressure at the stage’s inlet, $\alpha$ is gas void ratio, $q_m$, B/D, is mixture flow rate in the stage, and $q_{max}$, B/D, is the stage’s maximum flow rate at zero head. $K$, $E1$, $E2$, and $E3$ are coefficients as shown in Table 2. Eq. 4 is derived from the average data in a few stages as shown in Table 2, after using the correlation to get one stage head, just multiply the number of stages (6 or 8) to get the total head of the stages.

Lower pump compresses gas/liquid mixture and mixes gas and liquid. The lower pump should avoid unstable surging operation and gas lock. The critical model to judge pumping stability is

$$\left(\frac{q_m}{q_{max}}\right)_{crit} = K_e (p_{in})^{E4} \alpha^{E5}.$$  \hspace{1cm} (5)

Where $q_m$, B/D, is pumping mixture rate and $q_{max}$, B/D is the pump’s maximum flow rate where head is zero, $p_{in}$, psia, and $\alpha$ are the pressure and gas void ratio at pump intake. The above correlation works for each stage in a pump, but one may just use it for the first stage at the pump intake. If the first stage is stable all the stages above are in stable area.

During the design of a single or tapered pump, use Eq. 5 to judge the stability at the intake of the single pump or the bottom pump of the tapered pump. If it is unstable, one may select another pump, change pump depth or reduce DLR to satisfy the critical stability condition. Once the single pump or the bottom pump is stable, Eq. 4 can be used to calculate the total stages of the single pump, or the pressure and flow rate at the bottom pump’s discharge for the design of an upper pump.

Like regular pump, gas handler’s performance also depends on gas void ratio and pump intake pressure. Eqs. 4 and 5 come from experimental data of regular pumps. For a specially designed gas handler, Eqs. 4 and 5 may loss their accuracy, and one should consult gas handler’s manufacturer for its performance with gas void ratio and its critical condition.

No matter for regular pump or gas handler, unstable surging or gas lock will appear at high gas void ratio. For the situation of low pump intake pressure and high GOR, fluid may flow in slug pattern at pump intake. Pump or gas handler behavior will be unstable or even gas lock once gas slug comes to pump intake. Therefore, it is important to avoid slug flow at pump intake. Slug flow can be judged by flow pattern model presented by Barnea (1987).

Using the same well in Table 1 as an example, if consider gas effect on pump performance, the discharge total fluid rate of bottom Pump B (at stage 50) is 1112 B/D, which is out of the operating range of pump A (378-899 B/D). The design can only produce 433 STB/D liquid. To produce 608 STB/D liquid, 244 stages are needed for the pump A. Therefore, when taken into the effect of gas interference, the original design by homogeneous model has a lower life and lower production rate. Luckily, the pumping is stable from Eq. 5.

To make the discharge rate of Pump B in the operating range of pump A, 105 stages are needed for the bottom Pump B, and its discharge rate is 890 B/D which is in the range of pump A. For pump A, 188 stages are needed to pump a total liquid rate of 608 STB/D.

**Pump Optimization**

Horsepower consumption of an ESP pumping well takes the major part of daily cost. A pump’s hydraulic horsepower is

$$HP = \frac{qH\gamma}{136,000E}.$$  \hspace{1cm} (6)

Where $E$ is pump efficiency. Improve the efficiency will reduce required horsepower and so operating cost.

A pump consists of stages, and each stage has its own efficiency since flow rate reduces stage by stage due to pressure increase. The total horsepower of a pump with $n$ stages can be expressed as,

$$HP = \sum_{i=1}^{n} \frac{q_iH_i\gamma_i}{136,000E_i}.$$  \hspace{1cm} (7)

**Optimal Design for Single Pump.** An optimal design is to design an ESP well with high pump efficiency, so less brake horsepower is needed to pump well fluid at DLR. The best design is to make Eq. 7 get a minimum value. As shown in Fig. 1, generally speaking, for pumping pure water, a pump has the highest efficiency if pumping rate is at the BEP rate. Therefore, the best design for pumping water is to select a pump whose BEP is at or closest to the DLR. It should be noted that the efficiency at BEP may not be the highest efficiency for some pumps. However, we use the BEP as the most effective rate of a pump in the paper.
When there is no free gas in pumping fluid, the liquid volume decreases linearly as pressure increases as shown by the 100% Liquid line in Fig. 4. The average rate through the pump is at the middle stage of the pump. A design method is to select a pump with BEP at (or closest to) the average rate. For the design, almost half stages work higher than BEP and another half work lower than BEP. If the pump’s efficiency curve is symmetric at the pump’s BEP, the design of using average liquid rate is the most efficient one. The method is called average rate method in the paper.

If gas presents in a pumping fluid, depending on gas void ratio, the gas/liquid mixture rate may reduce a lot from pump intake to discharge as shown in Fig. 4 for High GOR curves. Volume reduction is different for each stage. Fluid volume reduces more severely at lower stages at pump intake and the reduction slows down for upper stages.

For the compression curve of High GOR in Fig. 4, the average rate is at 1032 B/D that corresponds to stage 34. If the pump’s BEP is at the average rate, 33 stages work above the BEP and 155 stages work below the BEP. Therefore, the average rate method to design an ESP is not an optimal method for gassy wells. An optimal design should move the discharge rate close to BEP rate and the intake rate away from BEP rate to make most stages work around the BEP.

The mathematical model to complete the optimal design of an ESP well is rather complex. It depends upon gas void ratio, water cut, formation volume factor, bubble point pressure, pressures at pump intake and discharge places, and the shapes of corrected pump efficiency curve and head curve.

In the industry, simple methods, using liquid rate at pump intake and using DLR, are widely used. For gassy wells, the two methods may not get high pumping efficiency. Since the actual pumping rate is the sum of liquid rate and gas rate, using liquid rate at surface or pump intake will select a low BEP pump.

The method of using average rate (liquid rate and gas rate) works for linear compression theoretically. It gives higher efficiency than that of using the two methods of liquid rate. However, as shown in Fig. 4, most stages work at rates close to the discharge rate. To obtain high pumping efficiency, the total fluid (water, oil and gas) rate at pump discharge should be used.

The method of using discharge total fluid rate is to select a pump whose BEP is just above or equal to the total rate at pump discharge. Certainly, if no such a pump is available and there is a pump has a BEP just below the total rate, one may use it. The new method makes most stages work close to BEP and therefore the whole pump has high efficiency. As discussed above, the total fluid rate at pump intake should be less than the upper end of the pump’s operating range.

For example, for the Well B in Table 3, select the Pump A in Fig. 1. The corrected pump BEP rate is 708 B/D and the low and high ends of its operating range are 400 and 950 B/D respectively. Assume the gas in the pump is only compressed and no gas separation at pump intake. At pump intake, pressure is 862 psia, liquid rate is 691 B/D, and total rate (gas and liquid) is 942 B/D. At pump discharge, pressure is 1246 psia, liquid rate is 686 B/D, and total rate (gas and liquid) is 857 B/D. The average total rate is 900 B/D.

The pump’s operating range (400 and 950 B/D) covers the total fluid rates at pump intake and discharge (942 B/D and 857 B/D) for the DLR, so the selection of Pump A is acceptable. At the pump intake, gas void ratio is 27% and pressure is 862 psia. From Eqs. 4 and 5, the pumping is stable and gas interference is negligible. Homogeneous model works fine for the case. The calculated required stages are 76 to produce the DLR. The efficiencies for all 76 stages are calculated and plotted on Fig. 7 for every 5 stages.

As shown in Fig. 7, the efficiency at BEP is 59%. All the stages work below the efficiency. The top stage (stage 76) at pump discharge has the largest efficiency of 56%. The first stage at pump intake gets the lowest efficiency of 50%. Obviously, if the top stage worked just below or at the BEP, all the stages would have higher efficiencies and the pump would work more efficiently.

Pump A has a BEP 708 B/D which is the closest to the liquid rate at pump intake of 691 B/D form all the pumps from a manufacturer. The pump is selected by the method of using the liquid rate at pump intake. From the proposed method of using total fluid rate (liquid rate and gas rate) at pump discharge, a pump with BEP at or just above 857 B/D should be used.

For the case, the method of using average fluid rate of 900 B/D will also give high pump efficiency. The method of using the DLR at 600 B/D will give a lowest efficiency pump for the case.

The above example assumes gas is just compressed in stages. If the gas solutes into the oil as pressure increases, the calculated stages are 79. And at pump intake, pressure is 862 psia, liquid rate is 691 B/D, and total rate is 942 B/D. At pump discharge, pressure is 1331 psia, liquid rate is 713 B/D, and total fluid rate is 768 B/D. The tap stage has the highest efficiency of 58% and the first stage has a lowest efficiency of 50%. Obviously, if the pump had a BEP of 768 B/D or a little bit above, the whole efficiencies would be improved.

Due to small compressibility of water, when pumping water the four methods (using DLR, liquid rate at pump intake, average total rate, and total rate at pump discharge) will give very similar results and therefore all of them give high efficiency. When pumping oil or oil with small amount of gas, except for the method of using DLR, all the other three methods will give similar high efficiency. For gassy wells, using the methods of using total rate at pump discharge and using average total rate yields high pumping efficiency.

**Optimal Design for Tapered Pump.** The optimal design of a tapered pump follows the same algorithm of obtaining a minimum value of Eq. 7 as discussed for a single pump. In addition to the shape and maximum value of each pump’s efficiency curve, the overlap size of the operating ranges of adjacent pumps is also a major factor for the optimization. The optimization process is complex and one should use computer program to complete it. Practically, simple methods are used.
As discussed above, the basic criterion of designing a tapered pump is that the operating ranges of adjacent pumps should overlap, the low operating end of its top pump should be smaller than the total fluid rate at the pump’s discharge, and the high operating end of its bottom pump should be greater than the total fluid rate at the pump’s intake.

In addition to the basic criterion, the efficiency of a tapered pump can be improved by simple methods of using total fluid rate at the place of pump discharge or using average total fluid rate. For tapered pump, only the total fluid rate at the top pump’s discharge is known before selecting pumps for a given DLR. All other discharge rates are calculated values by assuming stage number for each lower pump. The optimal design is a trial and error method by adjust pump stages to make each pump’s BEP is just higher or equals its discharge total rate, especially for the top pump which generally contains more stages. The bottom pump may be special and use average fluid rate method to design if it contains fewer stages.

For tapered pump design, the method of using average total rate is also a good method, and, for some cases, may even yield higher efficiency than that of using total discharge rate. The average total rate is not the rate of all pump stages in a tapered pump. Each pump should be designed by its own average total rate. Therefore, the procedure is also a trial and error method. As shown in Fig. 4, if cut the volume reduction curve into two or more sections (corresponding to two or more pumps), each section behaviors more linear and can be simulated with a straight line. As discussed above, the method of using average total rate will give an optimal pump if the volume reduction in stages is linear.

Same as design a single pump, pumping stability should be checked and performance degradation should be considered in stage by stage calculation. Generally, only the bottom pump may be affected by gas interference. Also, the flow pattern at the bottom pump’s intake should be calculated, and slug flow should be avoided.

For the same case of designing a tapered pump with gas interference, as discussed above, bottom pump is Pump B and has stages 105, the top bottom is Pump A and has stages 188. The total rate at top pump’s discharge is 710 B/D. The required power is 45.3 HP. The design satisfies the basic criterion. To get high pumping efficiency, use the discharge total rate 710 B/D to select a pump whose BEP is just above the total rate. The Pump C in Fig. 8 is selected. The pump’s corrected operating range is 521 B/D to 900 B/D and BEP is 718 B/D. The new tapered pump consists of Pump B and Pump C. The calculated results are: the bottom pump has 105 stages and discharge pressure of 485 psia. The top pump needs 179 stages. The new design requires 44.5 HP that is less than the original 45.3 HP, so the new design has higher pumping efficiency.

Conclusions
In addition to correct for viscosity, specific gravity and frequency of a pump’s standard pump curve, the existence of free gas will degrade pump performance and should also be corrected.

Basic criterion in selecting a tapered pump is that the high operating end of bottom pump should be greater than the total rate at the pump’s intake, the low operating end of top pump should be lower than the total rate at the pump’s discharge, and the operating ranges of adjacent pumps should overlap.

Tapered pump design is a process of trial and error. Except for top pump, all lower pumps need adjusting assumed stage number to satisfy each pump’s operating range and improve pumping efficiency.

Fluid rates in pump (single or tapered) reduce at lot stage by stage for gassy wells. A computer program can design a maximum efficiency pumping.

Two simple methods, using total rate at the place of pump discharge and using average total rate, can be used to design high efficiency pumping of gassy wells for both single and tapered pumps.

Nomenclature

\[ C_q, C_{16}, C_{E}, C_{HP} = \text{viscosity correlation factors} \]
\[ n = \text{number of stages} \]
\[ p = \text{pressure, psia} \]
\[ q = \text{volumetric flow rate, B/D} \]
\[ H = \text{head, ft} \]
\[ E = \text{efficiency, \%} \]
\[ HP = \text{horsepower, HP} \]
\[ K, E_1, E_2, E_3 = \text{regression constants} \]
\[ K_c, E_4, E_5 = \text{regression constants} \]
\[ N = \text{rotation speed, RPM} \]
\[ A = \text{gas void ratio} \]
\[ \gamma = \text{specific gravity} \]

Subscripts

\[ 1 = \text{initial condition} \]
\[ 2 = \text{final condition} \]
\[ crit = \text{critical condition for pumping stability} \]
\[ i = \text{stage number} \]
\[ in = \text{intake condition} \]
\[ m = \text{gas liquid mixture} \]
\[ max = \text{maximum value} \]
$w = \text{water}$

$\nu s = \text{viscous}$

**Acknowledgments**

The authors would like to thank IHS INC. for permission to publish this paper.

**References**


**SI Metric Conversion Factors**

<table>
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<tr>
<th>Unit</th>
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<tr>
<td>°API</td>
<td>141.5/(131.5+°API) = g/cm³</td>
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<tr>
<td>bbl</td>
<td>1.589 873 E-01 = m³</td>
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<tr>
<td>ft</td>
<td>3.048* E-01 = m</td>
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<tr>
<td>ft³</td>
<td>2.831 685 E-02 = m³</td>
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<td>°F</td>
<td>(°F-32)/1.8 = °C</td>
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<tr>
<td>hp</td>
<td>7.460 43 E-01 = kW</td>
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<tr>
<td>in.</td>
<td>2.54* E+00 = cm</td>
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<tr>
<td>lbm</td>
<td>4.535 924 E-01 = kg</td>
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*Conversion factor is exact.*
**Fig. 1**—Standard pump curve—Pump A.

**Fig. 2**—Corrected standard pump curve—Pump A.

**Fig. 3**—Pressure profile of Well A.

**Fig. 4**—Fluid volume reduction with stages.

**Fig. 5**—Homogeneous model on liquid rate.

**Fig. 6**—Standard pump curve—Pump B.
Table 1—Well A

<p>| | | | |</p>
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Table 2—Zhou and Sachdeva’s Coefficients

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Table 3—Well B

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