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Calculation and Implications of Breakdown Pressures in Directional Wellbore Stimulation

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Abstract

In 1898, Kirsch published equations describing the elastic stresses around a circular hole that are still used today in wellbore pressure breakdown calculations. These equations are standard instruments used in multiple areas of petroleum engineering, however, the original equations were developed strictly for vertical well settings. In today's common directional or horizontal well situations, the equations need adjusted for both deviation from the vertical plane and orientation to the maximum and minimum horizontal in-situ stress anisotropy. This paper provides the mathematical development of these modified breakdown equations, along with examples of the implications in varying strike-slip and pore pressure settings. These examples show conditions where it is not unusual for breakdown pressure gradients to exceed 1.0 psi/ft and describes why certain stages in "porpoising" horizontal wells experience extreme breakdown issues during hydraulic fracturing treatments. The paper also discusses how, in most directional situations, the wellbore will almost always fail initially in a longitudinal direction at the borehole wall, after which the far-field stresses will take over and transverse components can be developed. Tortuosity and near wellbore friction pressure can actually add to forcing the initiation of such longitudinal fractures, which can then have cascading effects on other growth parameters such as cluster-to-cluster and stage-to-stage stress shadowing. Special considerations for highly laminated anisotropic formations, where shear failure of the wellbore may precede or preclude tensile failure, are also introduced. Such failure behaviors have significant implications on near wellbore conductivity requirements and can also greatly impact well production and recovery efforts.

Introduction

In vertical wells it is relatively easy to accept that a tensile fracture will initiate along the axis of the well and propagate in the direction of maximum horizontal stress, opening perpendicular to the minimum stress. As the industry has increasingly moved to horizontal well completion and stimulation, the paradigm that fractures will orient in the plane of the maximum and intermediate stress of the earth tensor, and continue to open against the minimum horizontal stress, has been retained. Field-scale development plans are based on the expectation of nearly parallel vertical fractures that are orthogonal to the planned borehole (transverse), or set at a predictable angle to the well axis. Azimuth or strike of the fracture is expected to be given by the azimuth of maximum horizontal stress. When the horizontal stress anisotropy

is low, a more complex system of possibly orthogonal fractures, or some complex joint network, is expected.

This focus on the large-scale fracture morphology often ignores the fact that tensile hydraulic fractures must first initiate at the borehole wall. The stress concentration around the borehole is severe, and is affected by many parameters including orientation and magnitude of the stress tensor, borehole inclination and azimuth, pore pressure of the reservoir, rock mechanical properties, and internal wellbore pressure or mechanical bearing loads on the borehole surface. Breakdown conditions are affected by more than the stress state, and must also consider rock fabric conditions. This is especially true in anisotropic formations with laminated characteristics that may include discrete bedding planes, inter-bedded clay and ash layers, hard streaks such as anhydrite and carbonate stringers, and naturally fractured or jointed rock masses. Most theoretical analyses of borehole failure simplify the solution by considering only isotropic and homogeneous materials, often with the further simplification of linear and elastic deformation behavior (Britt and Smith, 2009).

Real reservoir rocks, especially in unconventional reservoir systems, are none of these things. It is often necessary to design and execute hydraulic fracture completions in wells drilled nearly parallel to bedding, in highly laminated and anisotropic formations, with frequent weak planes and where large contrasts in mechanical properties exist over very small vertical distances. This paper attempts to discuss the near-well conditions of borehole failure, especially tensile breakdown, considering the effects of a highly anisotropic and layered formation. A detailed numerical solution to the problem is not promised, and is probably unlikely to be valid when all the imponderable variables are considered. However, the hope is that this discussion will engender more focus on the connection between the well and the theoretical “far field” fractures than is represented by many fracture design simulators. The implications that such connections, or lack thereof, have on fracture propagation and follow-up production efficiency are also considered.

History of Kirsch Equations in Vertical Wells

In 1898, Kirsch published a solution for the stresses around a circular hole in an infinite linear-elastic plate, acted upon by orthogonal principal stresses (Eqs. 1-4). The solution is two dimensional and assumes plane-strain conditions in the plane of the plate. This solution has been widely accepted in the petroleum industry as adequate to represent the stresses around a borehole acted upon by far-field earth net stresses. The assumption of linear-elastic deformation is a limitation that is commonly overlooked for all wellbore orientations.

$$\sigma_r = \frac{\sigma_h + \sigma_H}{2} \left(1 - \frac{r_w^2}{r^2} \right) + \frac{\sigma_h - \sigma_H}{2} \left(1 - 4 \frac{r_w^2}{r^2} + 3 \frac{r_w^4}{r^4} \right) \cos 2\theta + \frac{r_w^2}{r^2} (P_w - \alpha_v P_o) \quad \text{Eq.1}$$

$$\sigma_t = \frac{\sigma_h + \sigma_H}{2} \left(1 + \frac{r_w^2}{r^2} \right) - \frac{\sigma_h - \sigma_H}{2} \left(1 + 3 \frac{r_w^4}{r^4} \right) \cos 2\theta - \frac{r_w^2}{r^2} (P_w - \alpha_v P_o) \quad \text{Eq.2}$$

$$\tau_{r\theta} = -\frac{\sigma_h - \sigma_H}{2} \left(1 + 2 \frac{r_w^2}{r^2} - 3 \frac{r_w^4}{r^4} \right) \sin 2\theta - \frac{r_w^2}{r^2} (P_w - \alpha_v P_o) \quad \text{Eq.3}$$

$$\sigma_v = P_{ob} - \alpha_v P_o \quad \text{Eq.4}$$

Where,

P_o = far-field pore pressure at true vertical depth, psi

P_{ob} = overburden pressure, psi

P_w = wellbore fluid pressure, psi

r = distance from wellbore, in

r_w = wellbore radius, in

α_v = Biot's coefficient in the vertical plane, unitless

σ_h = minimum horizontal in-situ net effective stress, psi

σ_H = maximum horizontal in-situ net effective stress, psi

σ_v = vertical net effective (intergranular) stress, psi

σ_r = radial net stress, psi

σ_t = tangential (hoop) net stress, psi

$\tau_{r\theta}$ = shear stress in the plane of the borehole stress, psi

θ = angle from direction of minimum in-situ stress, degrees

The prediction of radial, tangential (hoop stress), shear stress in the plane of the hole, and axial stress along the presumed borehole (normal to the plate) are given in Eqs. 1-4. The resultant stresses are net effective, or intergranular stresses, supported by the rock framework. In this application of the equations, the internal wellbore fluid pressure is assumed to be applied to the surface of the borehole, without penetration into the pore space of the rock. Breakdown of the well is assumed to occur when the internal wellbore pressure is high enough to set the tangential stress (σ_t) to zero at the angle around the well of minimum stress. The pressure may also have to exceed some intact rock tensile strength, if accounted for. For a non-penetrating fluid assumption, the axial stress (σ_v) is independent of wellbore fluid pressure and initiation of a fracture transverse to the wellbore is impossible.

Aadnoy (2008) has modified the breakdown prediction to account for the effect of Poisson's ratio on the interaction of stresses and strains in different orientations. His solution also makes the assumption of a non-penetrating fluid, but accounts for thermally induced stress changes around the borehole. For a penetrating fluid case, where the formation is permeable and the borehole face is not obstructed by mud cake, a transverse fracture can initiate if the pore pressure around the well exceeds the axial stress, as suggested by Grandi, Rao, and Toksoz (2002). This takes place where the axial stress is less than the minimum tangential net stress at the same wellbore pressure. This condition can also be achieved if the borehole intersects an open fracture transverse to the plane of the hole, which is similarly undamaged by drilling fines. Abbas and Lecampion (2013) gives a condition for transverse fracture initiation where a transverse notch must exist around the wellbore for fluid pressure to act upon.

An interesting laboratory and field study observed fracture initiation in vertical wells in the highly laminated Antrim Shale (Kim and Blaisdell, 1979). Their results suggest that transverse fractures are very difficult to induce, even in anisotropic, laminated systems with very weak bedding planes. Such observations make it hard to accept the routine expectation of induced transverse vertical fractures from horizontal wells in highly laminated systems, frequently with very adverse stress conditions around the well. With this in mind, a starting place is to extend the theoretical Kirsch equations, with all their limiting assumptions, to arbitrarily oriented wellbores in a three dimensional differential stress tensor as discussed in the next section.

Mathematical Development of Deviated Wellbore Breakdown Equations

A three-dimensional earth stress tensor can be initialized assuming a gravitationally dominant loading. In this case the vertical stress is given by Eq. 5 as related to the weight of the overburden. Net intergranular or effective stress is then given by the Terzaghi (1943) equation in Eq. 6. In this analysis, and for simplicity only, the Biot's (Biot and Willis, 1957) poroelastic constant in the vertical direction (α_v) is taken to be 1.0 in all further discussion. The minimum horizontal net stress is then derived assuming uniaxial vertical strain (Eq. 7). The maximum horizontal stress equation (Eq. 8) allows application of both lateral tectonic strain and a stress offset boundary condition. The effect of the stress and strain offsets on the minimum stress induced by Poisson's ratio, along with the assumed zero horizontal strain boundary condition, is ignored in this case, also for simplicity. The three principal stresses are oriented in space,

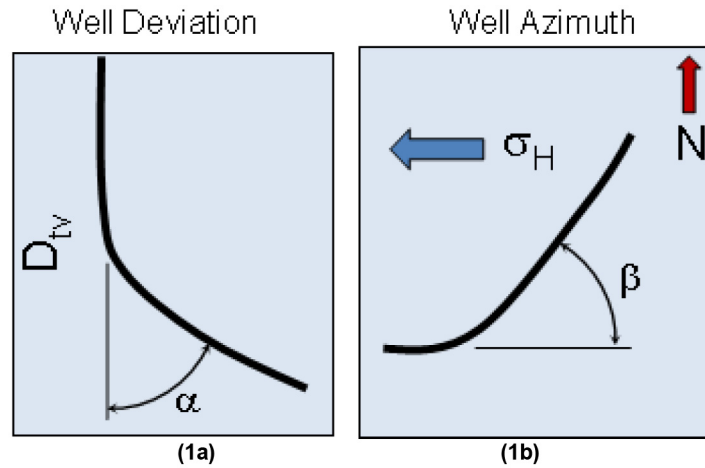


Figure 1—(1a) shows the angle, α , of the wellbore inclination as it deviates from the vertical plane. (1b) shows the azimuth, β , relative to the direction of maximum horizontal stress.

with the vertical stress assumed to be driven by gravity. In tectonically active areas where the maximum stress is in fact not vertical, the stress tensor can be rotated to any orientation, in which case all directional angles are relative to the axes of principal stress.

$$P_o = D_{tv}\gamma_p \quad \text{Eq.5}$$

$$\sigma_v = D_{tv}\gamma_{ob} - \alpha_v P_o \quad \text{Eq.6}$$

$$\sigma_h = \sigma_v \frac{\nu}{1-\nu} \quad \text{Eq.7}$$

$$\sigma_H = \sigma_h + \epsilon_x E + \sigma_{tect} \quad \text{Eq.8}$$

Where,

D_{tv} = true vertical depth, ft

E = Young's modulus, psi

ϵ_x = regional tectonic strain, microstrains

γ_{ob} = overburden pressure gradient, psi/ft

γ_p = pore pressure gradient, psi/ft

σ_{tect} = regional tectonic stress, psi

ν = Poisson's ratio, unitless

After defining the principal stress tensor, it must be mapped to the wellbore coordinate system. Fig. 1a shows the assumed wellbore inclination from vertical (α), and Fig. 1b the azimuth relative to the direction of maximum horizontal stress (β). Application of the appropriate directional sines and cosines, as shown in Eqs. 9-14, allow derivation of the stresses along the axis of the well (S_z), and orthogonal to the well axis (S_x and S_y). Since the well is not aligned with the principal stress tensor, the shear stresses in the xy, yz, and zx planes must also be accounted for (Eqs. 12-14).

$$\text{Transformed x-direction stress } (S_x): S_x = \sigma_H \sin(\beta)^2 + \sigma_h \cos(\beta)^2 \quad \text{Eq.9}$$

$$\text{Transformed y-direction stress } (S_y): S_y = \cos(\alpha)^2 (\sigma_H \cos(\beta)^2 + \sigma_h \sin(\beta)^2) + \sigma_v \sin(\alpha)^2 \quad \text{Eq.10}$$

$$\text{Transformed z-direction stress } (S_z): S_z = \sin(\alpha)^2 (\sigma_H \cos(\beta)^2 + \sigma_h \sin(\beta)^2) + \sigma_v \cos(\alpha)^2 \quad \text{Eq.11}$$

$$\text{Shear stress in x-y plane (S}_{xy}\text{): } S_{xy} = \cos(\alpha) \sin(\beta) \cos(\beta) (\sigma_H - \sigma_h) \quad \text{Eq.12}$$

$$\text{Shear Stress in y-z plane (S}_{yz}\text{): } S_{yz} = \sin(\alpha) \cos(\alpha) (\sigma_v - \sigma_H \cos(\beta)^2 - \sigma_h \sin(\beta)^2) \quad \text{Eq.13}$$

$$\text{Shear Stress in z-x plane (S}_{zx}\text{): } S_{zx} = \sin(\alpha) \sin(\beta) \cos(\beta) (\sigma_h - \sigma_H) \quad \text{Eq.14}$$

Finally the radial, tangential and axial stresses around any arbitrarily oriented segment of the wellbore can be defined by Eqs. 15-17. The variable λ denotes the angle around the circumference of the well, relative to the sides of the hole (where $\lambda=0$), as determined by the orientation of the wellbore element if rotated to the horizontal plane.

$$\text{Radial well stress (}\sigma_r\text{): } \sigma_r = P_w - P_o \quad \text{Eq.15}$$

$$\text{Tangential well stress (}\sigma_t\text{): } \sigma_t = S_x + S_y - 2(S_x - S_y) \cos(2\lambda) - 4S_{xy} \sin(2\lambda) - \sigma_r \quad \text{Eq.16}$$

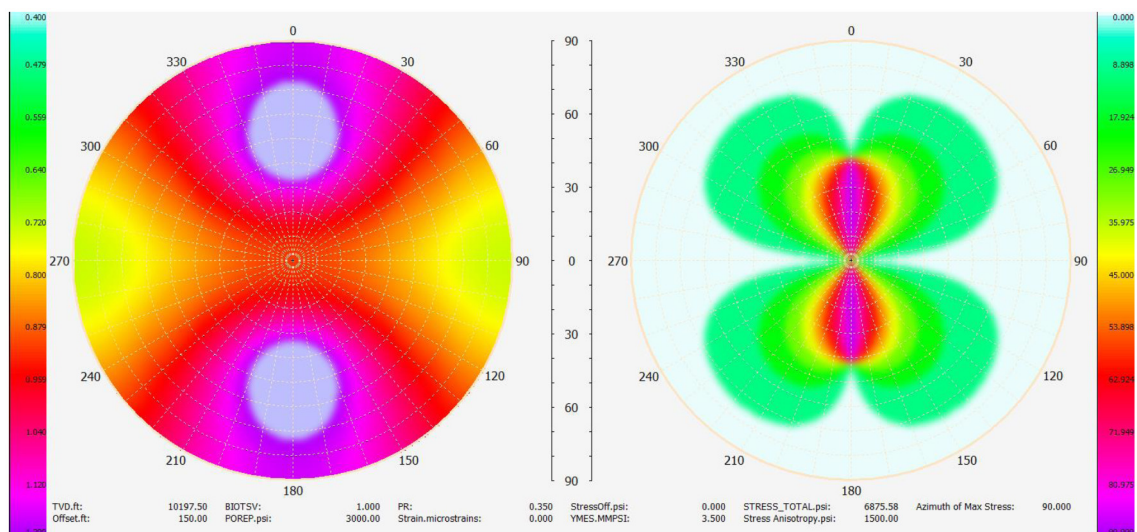
$$\text{Axial well stress (}\sigma_z\text{): } \sigma_z = S_z - 2\nu ((S_x - S_y) \cos(2\lambda) + 2S_{xy} \sin(2\lambda)) \quad \text{Eq.17}$$

Impacts on Breakdown Conditions – Pore Pressure, Poisson's Ratio, and Stress Differential

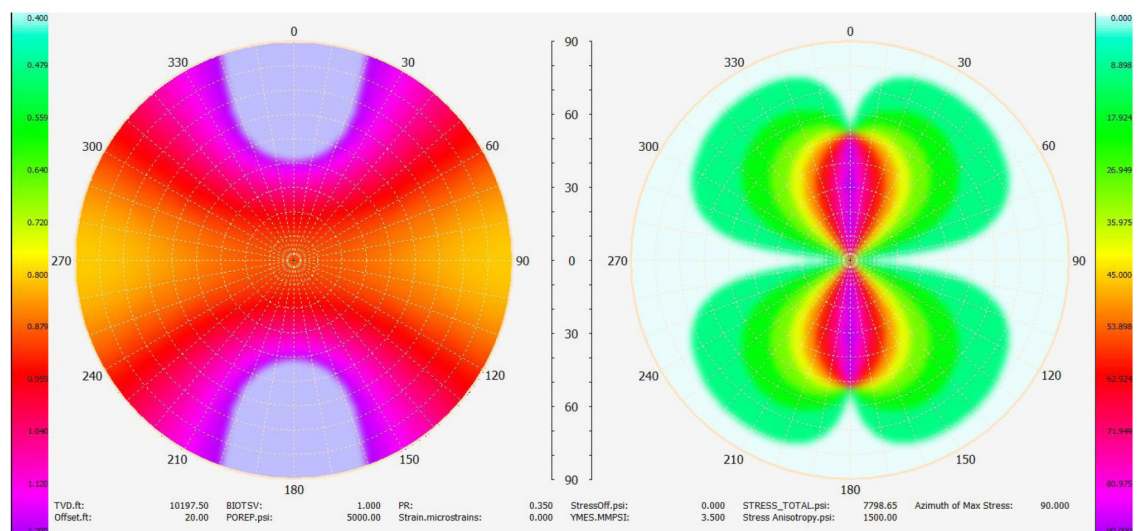
Eqs. 5-17 can be solved for varying conditions of pore pressure, Poisson's ratio (PR), and stress differential to determine the breakdown conditions for an intact section of borehole, with no existing natural fractures, shear planes, or borehole breakouts, for any angle of inclination from vertical and any azimuth relative to the direction of maximum horizontal stress. The plots shown in Figs. 2-4 demonstrate the results of these calculations for certain example conditions. There are two plots in each set of data presented in Figs. 2-4. The two plots are similar in the presentation of information. The compass bearings around the outer edge of each circle give the azimuth of a segment of the well (0° - 360°). For all cases shown, the azimuth of the assumed maximum horizontal stress is 90° - 270° , or east-west on the plots. The concentric circles in each plot show the inclination from vertical of the wellbore segment, in 10° increments. A vertical well falls at the center of each plot, whereas, horizontal wells, at 90° inclination, fall on the outer edge of each plot for any given azimuth. The color fill in each plot shows the magnitude of the parameter displayed by each plot with the scales to the left and right of the figures.

The plot on the left of each set of figures shows the breakdown gradient in psi/ft, for an assumed 1.0 psi/ft overburden gradient. This can also be interpreted as the breakdown pressure as a fraction of the overburden gradient. The color scale to the left of this plot gives the magnitude for each color, ranging from 0.4 to 1.2 psi/ft. The light blue-grey areas in these plots show conditions where bottomhole fluid pressure in the well must exceed overburden pressure by more than 20% to initiate breakdown. These sets of conditions, and the corresponding areas in purple in the adjacent plot, will present significant breakdown problems for those given well orientations. Since the wellbore pressure must exceed overburden, these areas may result in development of bedding-parallel shear failure planes and initiation of nearly horizontal fractures rather than vertical fractures. When these high stress areas fall in the build angle sections of the hole, such as inclinations of 30° - 70° from vertical, stuck pipe and hole collapse problems may be encountered if mud weight is not substantially increased.

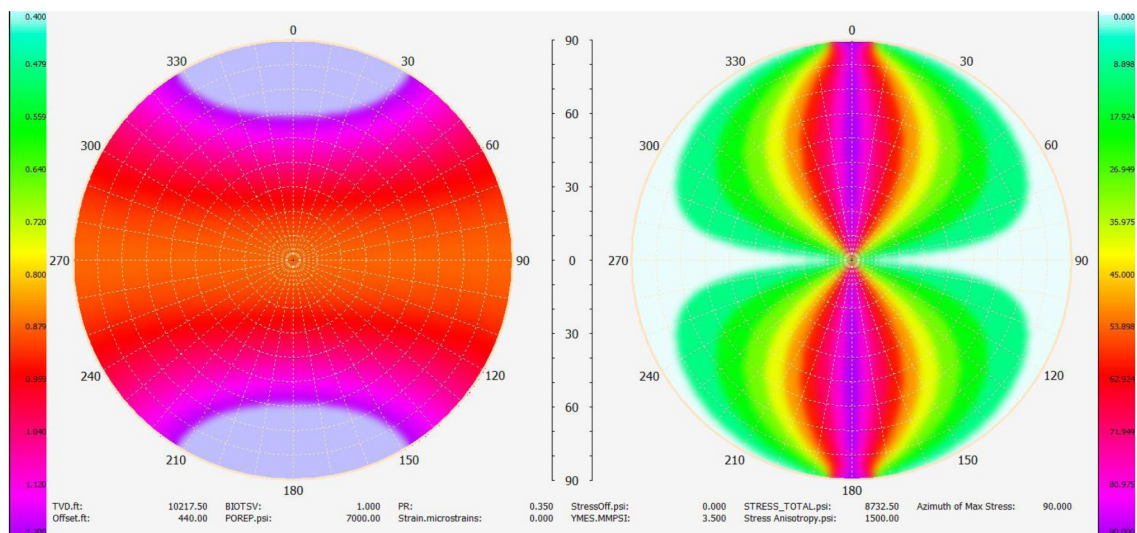
The right-hand plot in each set displays the orientation, or angle around the borehole circumference where the tangential stress is at a minimum. This represents the position around the hole where a tensile fracture is most likely to initiate. These conditions assume an isotropic medium with no pre-existing flaws. The color scale at the right ranges from 0° (in white), to 90° (in purple). The areas in white indicate fracture breakdown at the nominal top and bottom of the well, relative to the orientation for a horizontal



(2a)

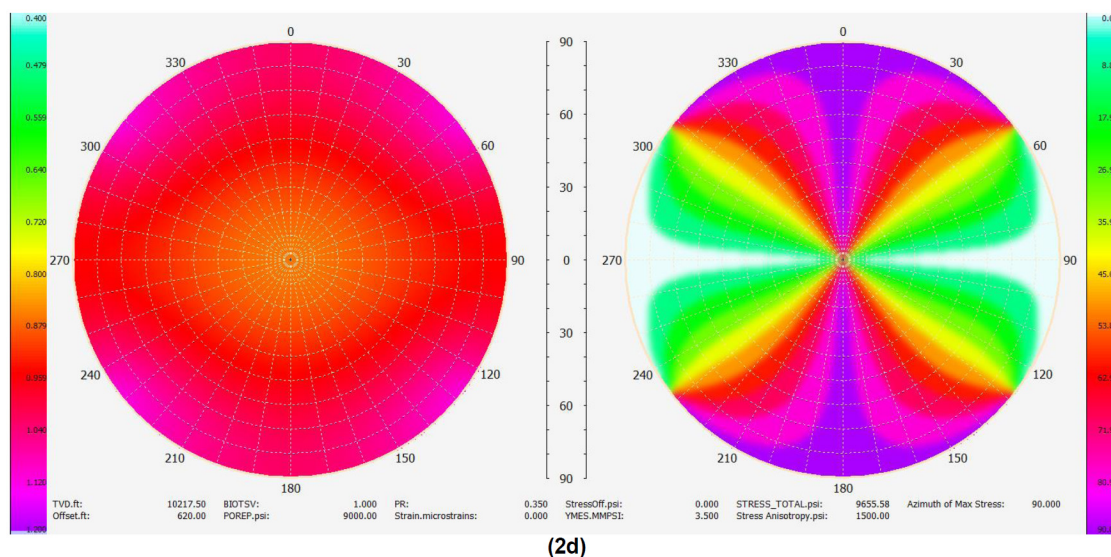


(2b)



(2c)

Figures 2a-2d—Breakdown pressure plots (left of each figure) and tensile fracture initiation angles (right of each figure) for 0.3 psi/ft (a), 0.5 psi/ft (b), 0.7 psi/ft (c), and 0.9 psi/ft (d) pore pressure gradients. An overburden gradient of 1.0 psi/ft, Poisson's ratio of 0.35, and a fixed x-y net stress differential of 1500 psi are assumed. Detailed descriptions of the plots are located in the paper text.



Figures 2a-2d— Continued.

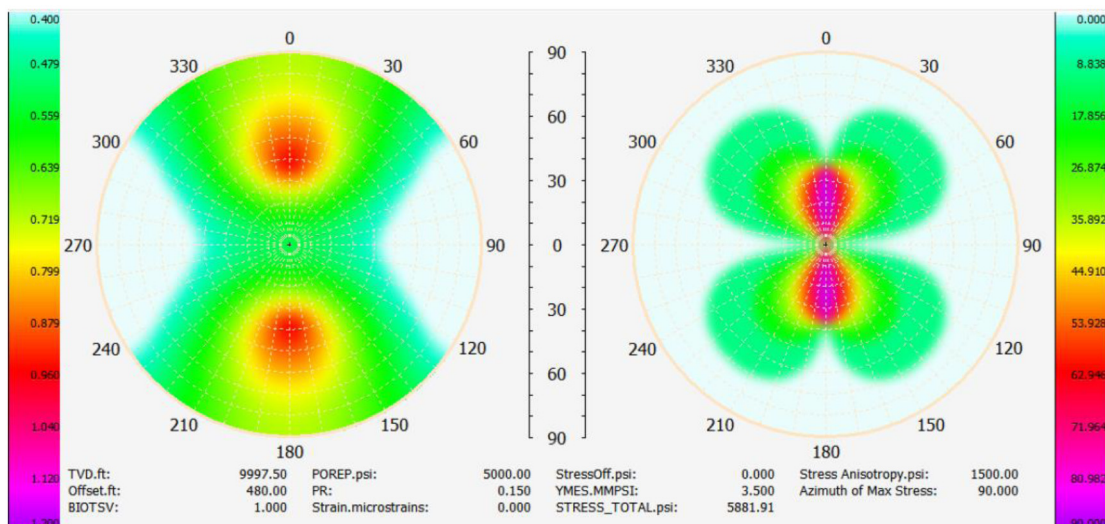
borehole. The green area shows breakdown in a range of angles from 10° - 30° off the top and bottom of the wellbore. These initiation angles can easily interact with weak bedding planes and alter the aperture and continuity of the fracture at the borehole face. The red areas show initiation angles from 50° - 80° off the top of the hole. A particular area of concern is around the point where the predicted breakdown angle changes from vertical to horizontal. Around this point the fracture is forced to make the most abrupt changes in direction relative to the well. This will usually result in very narrow fracture apertures, high friction pressure (tortuosity), and a high likelihood of early screenout at very low proppant concentration.

Pore Pressure Effects

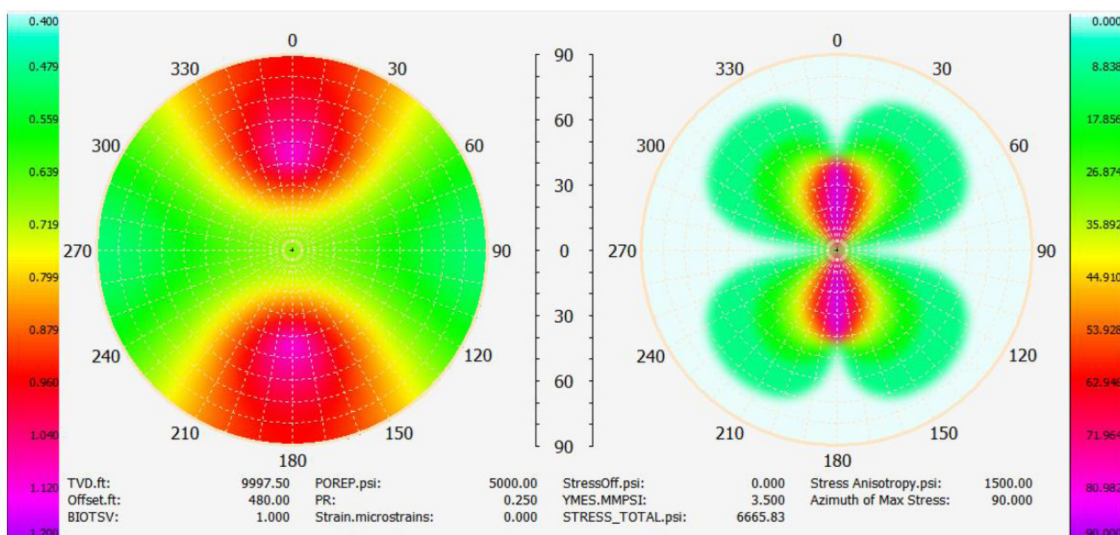
Figs. 2a-2d show the effect of varying reservoir pore pressure on these breakdown conditions, with all other parameters held constant. Using a typical shale Poisson's ratio of 0.35, with a fixed x-y net stress differential of 1500 psi, the breakdown pressure for any arbitrary well inclination and azimuth, relative to the direction of minimum horizontal stress, has been computed. The pore pressure gradient is varied in the figures with values of 0.3, 0.5, 0.7, and 0.9 psi/ft. A well drilled in the plane of the preferentially induced fracture will be east-west, at an azimuth of 90° - 270° , or in the direction of maximum horizontal stress. A well drilled north-south will generate transverse fractures, normal to the well axis for a horizontal completion, and drilled in the direction of minimum horizontal stress.

Fig. 2a demonstrates a 0.3 psi/ft pore pressure and shows a possible area of concern with wells drilled within $\pm 15^{\circ}$ of north-south, in the build angle section from 40° - 70° . These wells may be subject to stuck pipe or borehole collapse if mud weight is not adjusted appropriately. A horizontal well drilled north-south will require a breakdown pressure of almost 20% greater than that of overburden. The prediction for a perfectly isotropic material shows a vertical fracture initiation. Any laminations or rock fabric, including weak planes, will probably result in a horizontal fracture under these conditions.

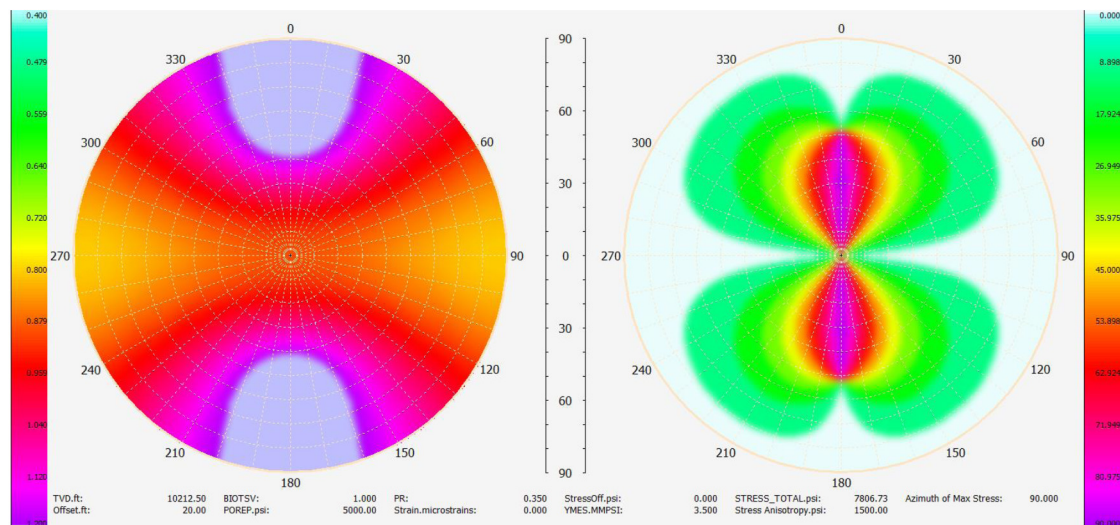
Fig. 2b shows the same results for a 0.5 psi/ft pore pressure. The colors in the breakdown plot show that the increased pore pressure translates to a higher breakdown pressure for all well conditions. The most severe breakdown conditions occur for wells drilled normal to the induced (or intended) fracture plane and at inclinations of more than 50° from vertical. While the plot at the right suggests that while vertical fractures are possible, they can be difficult to achieve in anisotropic, laminated systems when the wellbore fluid pressure is 25% greater than overburden.



(3a)

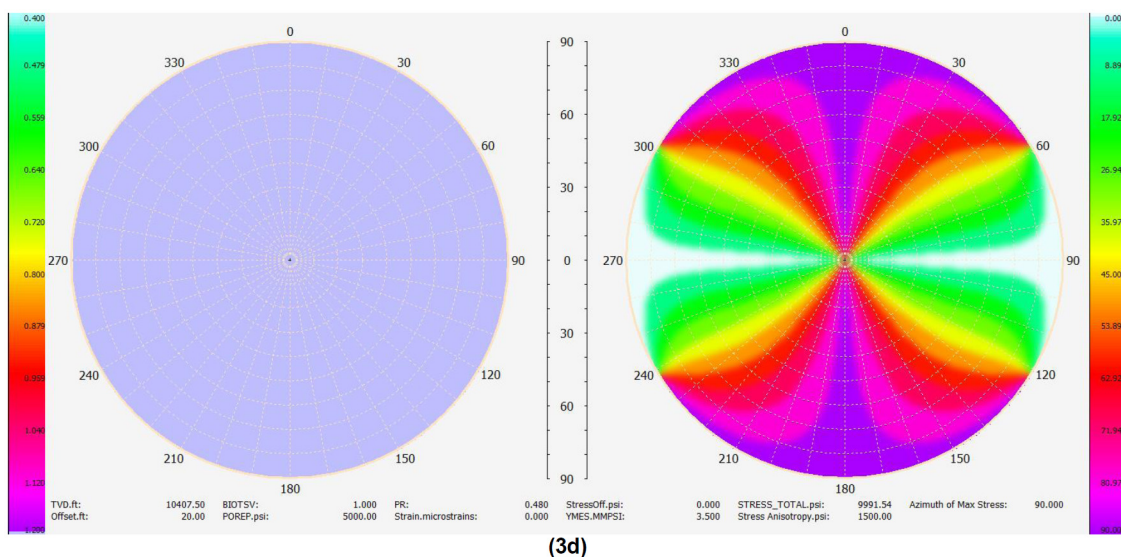


(3b)



(3c)

Figures 3a-3d—Breakdown pressure plots (left of each figure) and tensile fracture initiation angles (right of each figure) for Poisson's ratio values of 0.15 (a), 0.25 (b), 0.35 (c), and 0.48 (d). An overburden gradient of 1.0 psi/ft, a pore pressure gradient of 0.5 psi/ft, and a fixed x-y net stress differential of 1500 psi are assumed. Detailed descriptions of the plots are located in the paper text.



Figures 3a-3d— Continued.

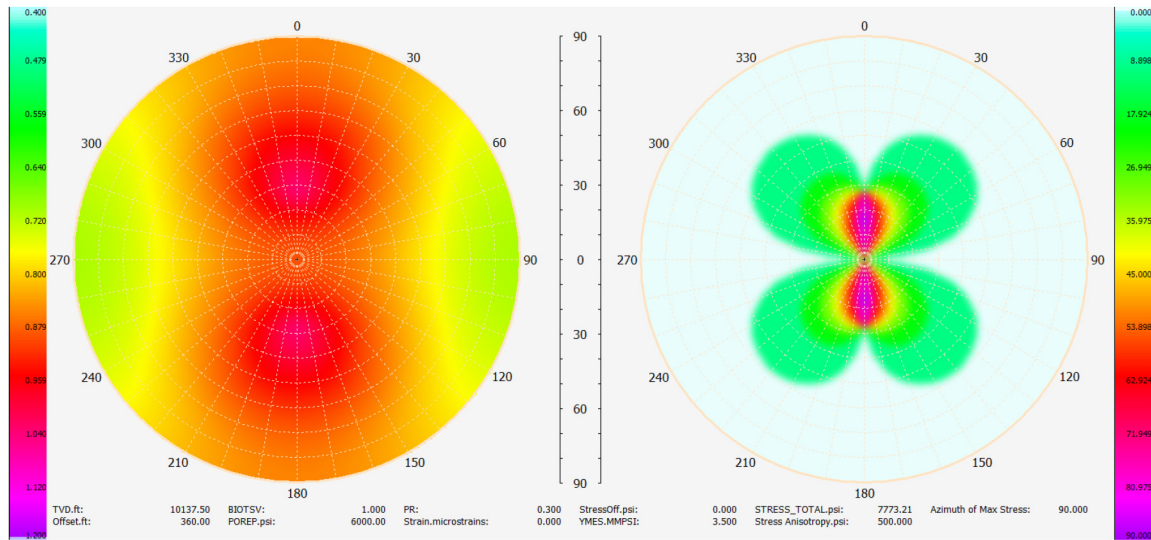
It is interesting to note that the breakdown pressure for a horizontal well drilled in the induced fracture plane, resulting in a longitudinal fracture, would break down at 80% of overburden pressure. A well drilled at 45° to the induced fracture plane would break down at 100% of overburden pressure for this case. Fig. 2c shows the results for a 0.7 psi/ft formation pore pressure. In this situation, a horizontal well drilled for transverse fracturing requires a breakdown pressure of more than 30% above overburden and is predicted to generate only horizontal or bedding parallel fractures. Overpressured formations in high differential stress regimes (strike-slip or reverse faulting) are extremely difficult to complete. In these conditions it may be impossible to generate vertical transverse fractures.

The last plot in the set, Fig. 2d, shows the results for a 0.9 psi/ft pore pressure case. This situation is interesting because the predicted breakdown pressure is actually lower than the previous case, but nearly insensitive to wellbore azimuth or inclination. With a 0.9 psi/ft pore pressure there is very little net stress remaining in the reservoir, and the system is essentially fluid supported. This means that all net stresses are nearly equal and close to zero. It may not be possible to sustain the assumed 1500 psi tectonic stress differential in this case because the internal friction in the rock is too low to support this shear stress. If it did occur for some reason, fractures initiated from horizontal wells in the range of up to 45° from the maximum stress direction would be horizontal or bedding parallel, with a breakdown pressure of only 1-10% above overburden. This low critical fracture gradient for generation of nearly horizontal fractures is consistent with observations presented by Barree, et al (2010).

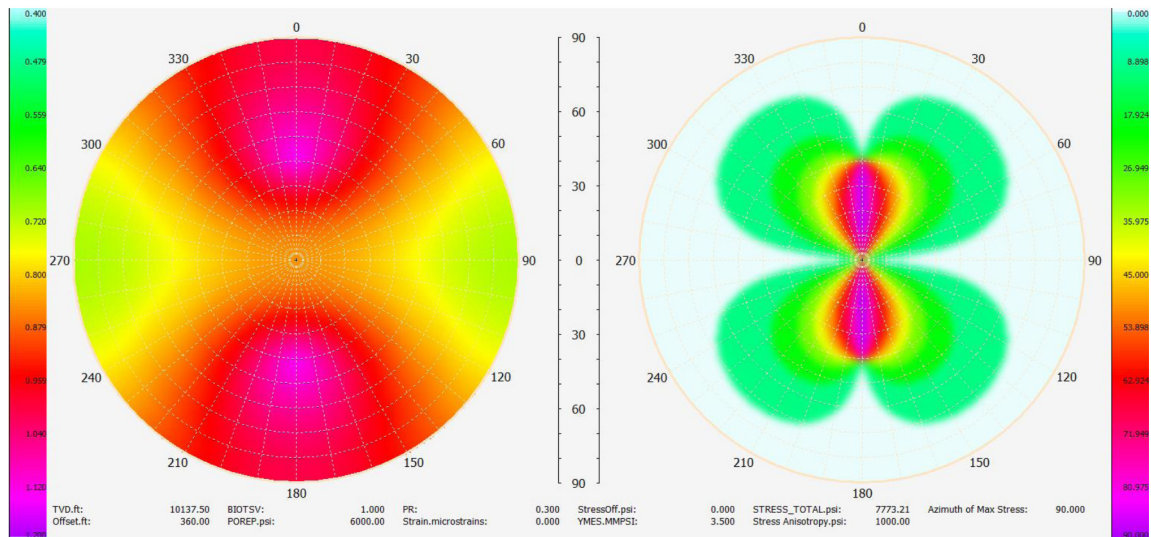
Poisson's Ratio (PR) Effects

Figs. 3a-3d show the effect of a varying Poisson's ratio (PR) for a normally pressured formation with a 0.5 psi/ft reservoir pressure gradient. The previous set, Figs. 2a-2d, showed a constant Poisson's ratio of 0.35, typical of a moderately high clay content shale. High clay content, or high Poisson's ratio formations, are difficult to fracture and are generally avoided. These tend to be described as "ductile" based on sonic log response, even though laboratory testing of these rocks shows they almost always exhibit brittle failure.

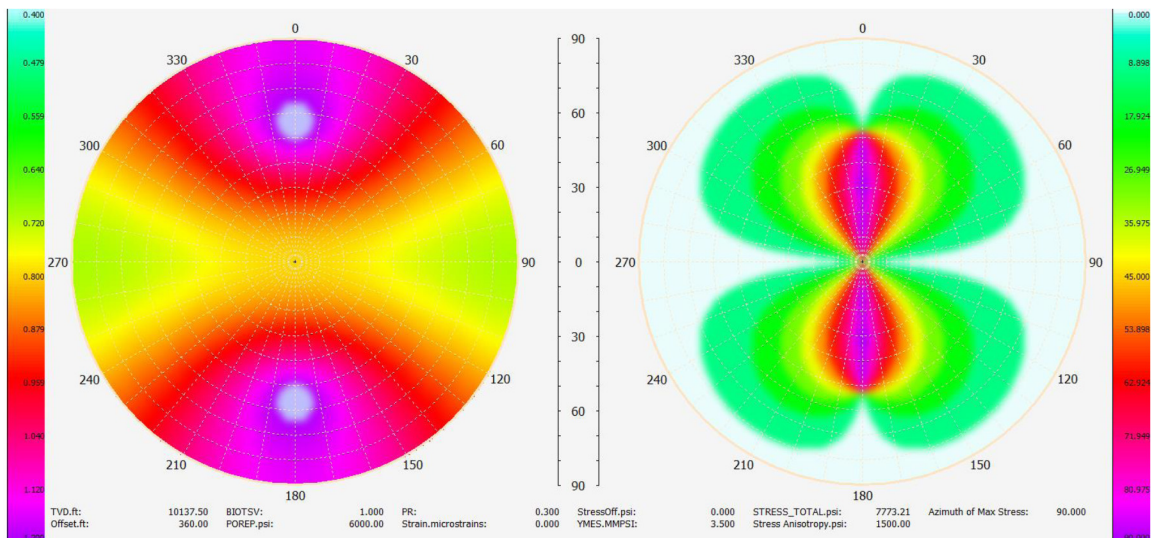
Figs. 3a-3d show the breakdown pressure gradient for any well inclination and azimuth for the assumed condition of a constant pore pressure gradient of 0.5 psi/ft at 10,000' TVD (net vertical stress of 5000 psi in normally pressured formation with Biot's constant = 1). The value of Poisson's ratio is varied over the range of 0.15, 0.25, 0.35, and 0.48. The lowest value is typical of a very clean sandstone, or "best"



(4a)

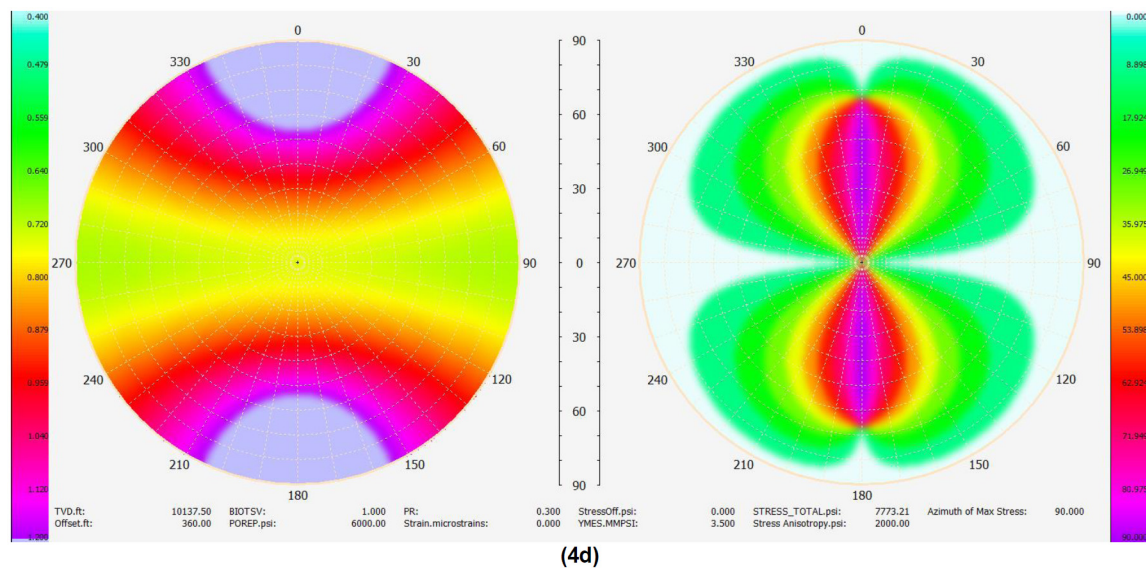


(4b)



(4c)

Figures 4a-4d—Breakdown pressure plots (left of each figure) and tensile fracture initiation angles (right of each figure) for 500 psi (a), 1000 psi (b), 1500 psi (c), and 2000 psi (d) horizontal differential stresses. An overburden gradient of 1.0 psi/ft, PR of 0.30, depth of 10,000 ft and a pore pressure gradient of 0.6 psi/ft are assumed. Detailed descriptions of the plots are located in the paper text.



Figures 4a-4d— Continued.

reservoir quality rock. The 0.25 case is typical of a sandstone or siltstone reservoir of low clay content. The case of Poisson's ratio of 0.48 is included to demonstrate the effect of dynamic undrained loading (Skempton's coefficient approaching 1) (Skempton, 1954) around a rapidly deformed borehole, as discussed by Barree, et al (2010). For these cases the horizontal stress differential was held constant at 1500 psi.

Fig. 3a, where $PR = 0.15$, presents problems for drillers when the well is oriented east-west, or in the longitudinal fracture plane. The large white areas, past an inclination of 45° , show a predicted fracture breakdown pressure less than the hydrostatic head of fresh water. These wells will fracture on a vacuum and cannot be drilled with a water-based mud system while maintaining returns. Any fracture breakdown conditions are easy, with the highest breakdown pressure at 97% of overburden at an inclination of 38° from vertical on a north-south azimuth (red zones). Horizontal wells drilled normal to the fracture plane will break down at 70% of overburden. For this reason, formations with low Poisson's ratio values are preferred for landing zones. As these respond with fast acoustic velocity, they are considered to be "brittle", which is in-fact a misnomer and is often misnamed as a "brittleness factor" by various logging and mineralogy-based techniques.

Figs. 3b and 3c show trends similar to Fig. 3a, but Fig. 3d deserves special attention. This case, with $PR = 0.48$, is not likely to be encountered under normal conditions. However, it can be generated dynamically by inducing rapid deformation around the wellbore in a very low permeability formation, especially one with low compressibility pore fluids (liquid). When the rock volume around the well is deformed by an imposed sandface pressure, the pore volume and bulk volume of the rock immediately adjacent to the borehole face is decreased. This causes an increase in the pore pressure which then must bleed off over some characteristic time, due to fluid mobility. If the fluid cannot move easily, the pore pressure can approach the applied wellbore pressure, which implies a Skempton's coefficient of 1. As the pore pressure increases, all net stresses are reduced, the system becomes fluid supported, and the effective undrained Poisson's ratio approaches 0.5. This can lead to either failure along bedding planes or failure of transverse fracture planes. The mode of failure is realistically determined by the rock fabric, localized planes or weakness, directional permeability, and other factors that make the outcome appear random. In the case shown, for ideal isotropic rock, the predicted breakdown gradient ranges from 30 to 50% greater than overburden. Low angle, bedding parallel or near horizontal fractures are predicted over most angles

of inclination for azimuths within 45° of the minimum horizontal stress. This refers to angles between the well and the induced fracture plane of up to 45° .

Differential Stress Effects

Figs. 4a-4d are presented to show the impact of differential stress on a formation of Poisson's ratio = 0.3 and a pore pressure gradient of 0.6 psi/ft. Assuming the existence of a cohesionless pre-existing fracture plane, or rock joint, with a 30° friction angle, the maximum differential stress that can be supported without shear slippage is approximately 1000 psi. The cases shown vary the horizontal differential stress over the range of 500, 1000, 1500, and 2000 psi to examine the differences in breakdown character for a jointed and intact rock from a nearly isotropic stress state to a reverse faulting condition.

Fig. 4a is the lowest differential stress case, with only 500 psi horizontal stress anisotropy. Low stress differential leads to benign treating conditions. A vertical well in this case breaks down at 89% of overburden, while a horizontal well drilled normal to the induced fracture planes breaks down at 84% of overburden. A horizontal well drilled for longitudinal fracture orientation breaks down at 70% of overburden. In this condition, a well can be drilled and hydraulically fractured at almost any inclination or azimuth. However, a different concern should be considered. With only 500 psi stress differential, a net fracture extension pressure of more than 500 psi places the fluid pressure in the fracture above both the minimum and maximum horizontal stresses. In this case the fracture may lose directional control and can begin to follow any paths of weakness in the rock dictated by rock fabric.

In this low differential stress condition, the stress concentration around the wellbore also becomes significant. At a true vertical depth (TVD) of 10,000 feet, as in this example, the minimum in-situ total stress is 7875 psi with a maximum horizontal stress of 8375 psi. Breakdown pressure for a horizontal well transverse to the intended fracture plane is about 8450 psi which is higher than both horizontal stresses. With an intact borehole, with no existing transverse natural fractures and a drilling mud cake, the well will preferentially break down along the axis of the borehole at the top and bottom of the well, generating a longitudinal fracture. Fluid pressure supplied to this fracture is sufficient to overcome the maximum horizontal stress and propagate the longitudinal fracture along the well. Presumably, the lateral deformation in the rock mass associated with the opening of the longitudinal fracture will generate some shear failures. These shear planes, at some angle to the initial fracture, will be invaded by fluid and will allow fractures to grow in a lower stress direction. Continued injection into these fracture must be supplied with pressure from the well sufficient to overcome the near-well stress and maintain sufficient aperture to place proppant.

So, while low stress differential makes breakdown easy, and can generate fracture "complexity" by allowing fractures to grow in different orientations, it is not without some potential problems. The stress concentration around the borehole promotes the initiation of longitudinal fractures. Any additional frictional pressure drop not associated with orifice pressure drops through the pipe (perforations, frac ports, sleeves, etc.) and generally considered to be "tortuosity" implies that the frictional pressure drop is taken at the sandface or the entry to the propagating fractures. This means that any tortuosity pressure drop provides additional fluid pressure applied to the surface of the borehole but is not transmitted into the body of the fractures at some distance from the well (of maybe as little as a few inches). This implies that completions exhibiting a tortuosity pressure drop equal to or greater than the in-situ stress differential can induce preferential longitudinal fractures instead of transverse fractures. In fact, the only way it appears that transverse fractures can be initiated is for existing transverse fractures to be intersected by the well, or to have pore fluid pressure around the well exceed the axial stress and rock strength before the tangential stress reaches the tensile failure point.

As the horizontal stress difference increases, through Figs. 4b-4d in the series, the point of maximum breakdown pressure migrates along the transverse well path to higher inclination angles. With high stress differential (Fig. 4d), the most difficult breakdown condition is a horizontal well drilled normal to the

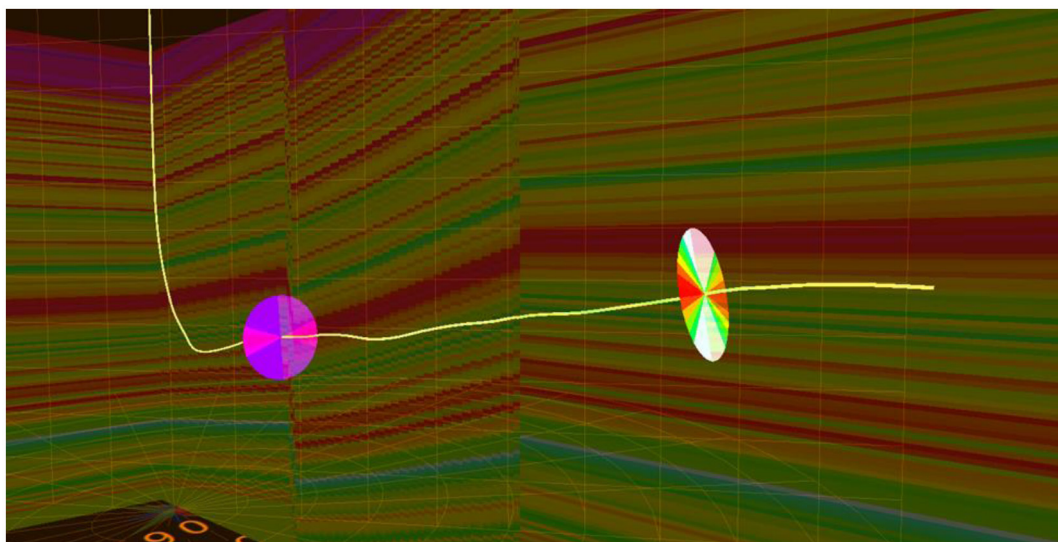


Figure 5—Graphical display of the tangential stress around the well at two potential fracture initiation points in a horizontal well system. The yellow trace is the wellbore. The background colors represent the minimum in-situ stress at various depths (“hot colors” are higher stress intervals). Pore pressure gradient is 0.92 psi/ft with a 2000 psi horizontal stress anisotropy. At the perforation location at the left of the figure, the breakdown pressure is 19,390 psi at the sides of the hole (pink colors) and 20,560 psi at the top and bottom of the hole (purple colors). The breakdown conditions for the perforation location at the right side of the figure are 17,180 psi fluid pressure at the top and bottom of the hole (white colors), and 19,060 psi at the sides of the hole (red colors).

expected fracture plane. This yields a breakdown gradient of 28% above overburden pressure. For the case of 10,000 foot TVD, this is a breakdown pressure of 12,780 psi. Minimum in-situ closure stress is still 7875 psi but the maximum closure stress is now 9875 psi. In this case the wellbore pressure at breakdown, for an intact borehole, exceeds the maximum horizontal stress by 2900 psi and exceeds the overburden stress by 2780 psi. Failure of the formation will probably be controlled by rock fabric in this case, leading to bedding parallel fractures, or fractures following any existing planes of weakness in the rock mass.

Implications of Tangential Stress Variation

The variation in tangential, or hoop stress around the circumference of the wellbore controls the orientation of fracture initiation and the wellbore pressure required to break down the formation. Figs. 2-4 show the minimum breakdown pressure for any well inclination and azimuth under various in-situ stress and rock property conditions. It is also worth considering the magnitude of the variation in tangential stress around the well. Fig. 5 graphically displays the tangential stress around the well for a horizontal well in a highly overpressured (0.92 psi/ft pore pressure gradient) reservoir with high horizontal stress anisotropy (2000 psi). These conditions are extreme, and are selected to stress the point that tangential stress magnitude and orientation can vary significantly along the same well, based on the landing point of each fracture initiation site.

At the perforation location at the left of the figure, the breakdown pressure is 19,390 psi at the sides of the borehole (pink colors) and 20,560 psi at the top and bottom of the borehole (purple colors). If this perforation location could be broken down within the limits of surface treating pressure constraints, the result would almost certainly be a horizontal fracture. When the weakness of the bedding planes and material strength anisotropy are considered, it is likely that bedding parallel shear, followed by fluid invasion along the shear planes, will result in a bedding-parallel failure condition at even less severe stress conditions than this. The breakdown conditions for the perforation location at the right side of the figure are 17,180 psi fluid pressure at the top and bottom of the hole (white colors), and 19,060 psi at the sides of the hole (red colors). This example clearly illustrates the importance of the vertical position of the horizontal well relative to the vertical variation in rock properties and stresses over the target pay interval.

Slight errors in geosteering, or neglecting relatively thin beds (the size of the borehole diameter or less) of different mechanical properties can lead to some unpleasant surprises regarding different breakdown conditions at each completion interval.

The difference in breakdown pressure for the second perforation interval is almost 2000 psi from the top of the hole to the sides. This also has some bearing on selection of perforation phasing for horizontal well stimulation. Perforations shot in the 90° arc at the sides of the hole are unlikely to ever break down under normal conditions. The fracture is forced to initiate at the top and bottom of the well, and may propagate along the well axis for some distance before turning, probably through a series of shear fractures, into the maximum stress orientation. The minimum in-situ closure stress at this point is 13,500 psi, along the axis of the well. If fluid pressure could act on the face of an existing transverse fracture, or could be applied to the pore space around the well, a transverse fracture may initiate well below the pressure required for even the lowest stress tangential breakdown.

Effects of Rock Fabric

The impact of rock fabric and anisotropy has been mentioned several times in this discussion. Finding or deriving borehole tensile and shear failure conditions under internal pressure has proven to be very difficult. An extensive literature search has not provided any satisfactory solutions, so a general discussion is proposed here. Most of the solutions typically applied in the petroleum industry to predict breakdown of a borehole assume linear elastic deformation of an isotropic and homogeneous medium. Numerous discussions of anisotropic borehole failure in collapse, under compressive loads have been published ([Shamsuzzoha 2011](#); [Papanastasiou and Zervos 2004](#); and [Papanastasiou and Thiercelin, 2011](#)) but rigorous solutions for tensile failure under internal pressure are not readily found. Other works focus on the reactivation of existing fractures in the far-field ([Williams-Stroud, et al 2012](#)) but fail to address the near-well conditions.

It is very likely that the initial conditions of failure at the borehole contribute to many of the observed problems in hydraulic fracture treatments of horizontal wells. If the fracture at the wellbore sandface is of small aperture, is composed of en-echelon shear fractures, or is oriented in an adverse stress direction, the near-well fracture geometry could lead to very high treating pressures (tortuosity friction) and rapid screenouts, even with low concentrations of small particles. To demonstrate the impact of material fabric, two breakdown tests were conducted on similar samples with drilled holes, internally pressurized with viscous fluid to the point of breakdown. In [Fig. 6](#), the sample on the left is nearly isotropic and homogeneous. The sample on the right is composed of the same material but is comprised of a stack of thinly laminated beds. Both samples were subjected to high confining stress, with the axis of maximum stress shown by the blue arrows. This stress represents the vertical, or overburden stress for a horizontal well. Note that the visible open “horizontal” lamination in the sample at right occurred after relieving the applied confining stress following the breakdown experiment.

The sample on the left of [Fig. 6](#) fails approximately as predicted by the conventional theory and as represented by the Kirsch equations. A tensile planar fracture is generated along the axis of the borehole, with breakdown in the direction of maximum stress. The sample on the right, comprised of a highly laminated composite with “bedding” laminations parallel to the borehole, fails in a much more complex mode. An axial, or longitudinal fracture forms along the top and bottom of the borehole, in the direction of maximum stress. A second dominant fracture propagates along the weak laminations, approximately along the centerline of the borehole. Secondary fractures emanate at angles to the borehole, and appear to partially follow the laminations in the material.

It is also worth noting that the stress along the axis of the borehole, and normal to the borehole, what would be considered the two horizontal stresses, are both very small compared to the induced “overburden” stress. Even in this highly exaggerated stress field, the breakdown follows the wellbore axis and is more affected by the material fabric than by the stress field. Also note the sizes of the final boreholes in

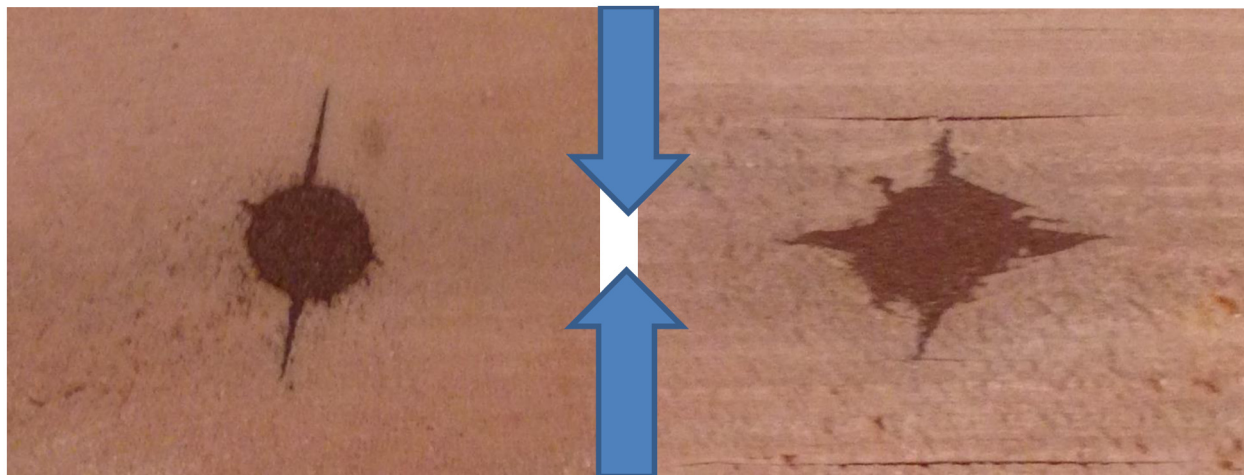


Figure 6—The pictures show two breakdown tests conducted on similar samples with drilled holes that were internally pressurized with viscous fluid to the point of breakdown. The blue arrows represent the maximum stress direction. The sample on the left fails as predicted by conventional theory, with a single planar fracture. The sample on the right, which was composed of a highly laminated system, demonstrates the complexity and associated issues that can occur in such a material. An axial, or longitudinal fracture forms along the top and bottom of the borehole, in the direction of maximum stress. A second dominant fracture propagates along the weak laminations, approximately along the centerline of the borehole. Finally, secondary fractures emanate at angles to the borehole, and appear to partially follow the laminations in the material.

the two cases. Both drilled holes were the same diameter prior to injection and breakdown. The final borehole diameter on the right indicates probable plastic deformation of the hole under internal load, as discussed by [Papanastasiou and Zervos \(2004\)](#). Even this simple experiment suggests that failure of a pressurized horizontal borehole in a highly anisotropic medium may be much more complex than predicted by simple linear-elastic homogeneous theories. Taking all the conditions discussed here together, it seems that longitudinal failure along a borehole, probably more controlled by rock fabric than stress, is much more likely than initiation of a planar tensile fracture transverse to the wellbore in practically any real-world situation.

Impacts on Treatment Design and Analysis

The breakdown and fracture orientation behaviors discussed thus far obviously have an impact on fracture initiation and propagation. As such, they also have impacts on fracture design and various treatment parameters. Although the conditions shown and described in [Fig. 5](#) are extreme, they provide an indication of the magnitude that breakdown pressure can approach, along with the variation in breakdown pressure that can exist around the wellbore within a given completion interval. This can have an impact on casing design and treatment pressure limitations. In a situation such as described in the heel perf set in [Fig. 5](#), it is unlikely that breakdown will ever occur. However, under even less extreme conditions, the required breakdown pressure may approach a level that is above the burst ratings of the installed casing system, which may not necessarily be designed with such situations in mind. This can lead to the inability of being able to initiate a fracture in a given cluster or stage, as the well “porpoises” through various inclination/azimuth combinations. This can result in skipping that stage and leaving untreated sections of the wellbore.

The near-wellbore tortuosity components that can be generated in the various situations discussed throughout this paper can have a significant effect on the actual fracturing treatment. The tortuosity and associated near well stress concentration can cause narrowing of the fracture aperture near the wellbore exit point that can lead directly and quickly to screenout. Even before the main fracturing treatment, this tortuosity can have an impact on pre-treatment pump-ins such as diagnostic fracture injection tests (DFIT’s). As discussed in Barree, [Miskimins and Gilbert \(2014\)](#), it is common to see high tortuosity values associated with horizontal well toe-stage DFIT’s. Such situations frequently occur in highly

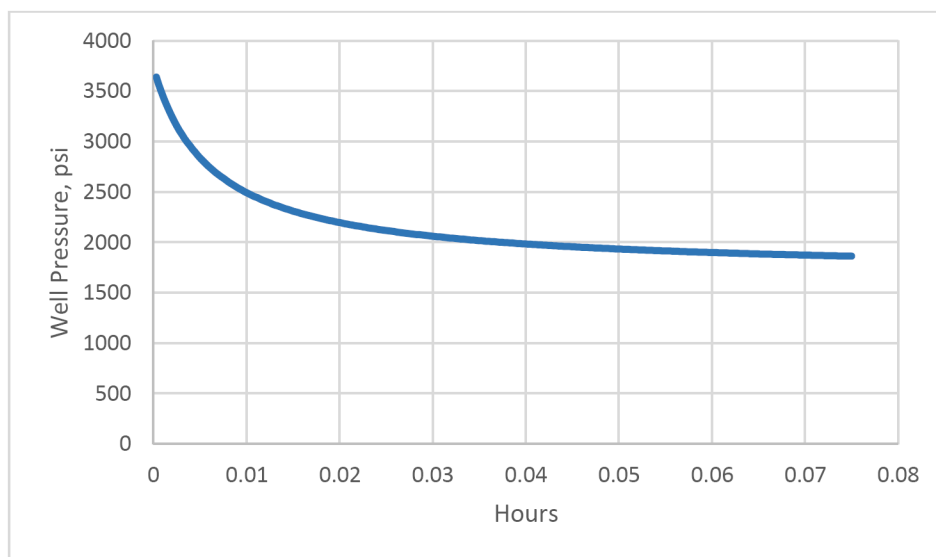


Figure 7—Post-shutdown pressure curve from a DFIT test. The tortuosity in this example is 970 psi/sqrt(BPM), which is of a moderate level. This tortuosity prevents a sharp ISIP from occurring, which leads to issues in determining fracture gradient.

laminated shale reservoir systems or other systems that are subject to shear failures. These high tortuosity values, which are likely generated by complex, near-wellbore fracture systems such as shown on the right side of Fig. 6, have a significant impact on the pressure after injection is terminated. Fig. 7 shows a slow pressure bleed-off exhibited after shut-down of the DFIT test. This slow bleedoff, through the near-wellbore tortuous path, makes it extremely difficult to determine an instantaneous shut-in pressure (ISIP) and consequently a fracture gradient. The inaccuracy of these two values then cascades into the rest of the DFIT analysis and can lead to inaccurate determination of flow regimes and the associated outputs such as pressure zone stress, pore pressure, and transmissibility.

High tortuosity, in the same magnitude as the x-y stress differential, can also increase the likelihood of longitudinal fracture initiation rather than transverse fractures. In any case where the injection pressure applied to the borehole wall, or net fracture extension pressure exceeds the horizontal stress anisotropy, a longitudinal fracture component is likely to initiate prior to a transverse fracture. The impacts of the longitudinal components of a fracture, which will occur in many situations as noted in Figs. 2-4, are far-reaching. These components of the fracture may be short or run along a significant portion of the wellbore, however, they are almost impossible to eliminate completely. Fig. 8 shows a picture of the final proppant concentration for a five cluster perforation stage, where the well azimuth and stress field are situated to grow transverse fractures at a 90° angle from the wellbore. The depth of the well is 13,500 ft with a fissure opening pressure of approximately 450 psi and a minimum in-situ stress of approximately 13,300 psi. Under these circumstances, a significant longitudinal fracture component is generated and runs almost 2000 ft along the wellbore.

The behavior seen in Fig. 8 has a large impact on the concept of stage diversion. This example shows one impact of the longitudinal fracture on the effectiveness of multiple-cluster limited-entry control of fracture initiation sites. For a cemented liner completion, the limited perforations generate a pressure drop from inside the pipe to the cement-formation annulus. The high tangential stress around the wellbore requires the fluid pressure on the borehole face to exceed the radial stress prior to fracture initiation. This causes an expansion of the borehole, opening a fluid filled annulus around the cement sheath. Once the longitudinal fracture initiates, fluid pressure from the annulus can travel along the well through the longitudinal fracture plane with minimal pressure gradient. Fluid from this fracture plane can access any transverse fracture planes, rock joints, and fissures that may exist or initiate through induced shear failure

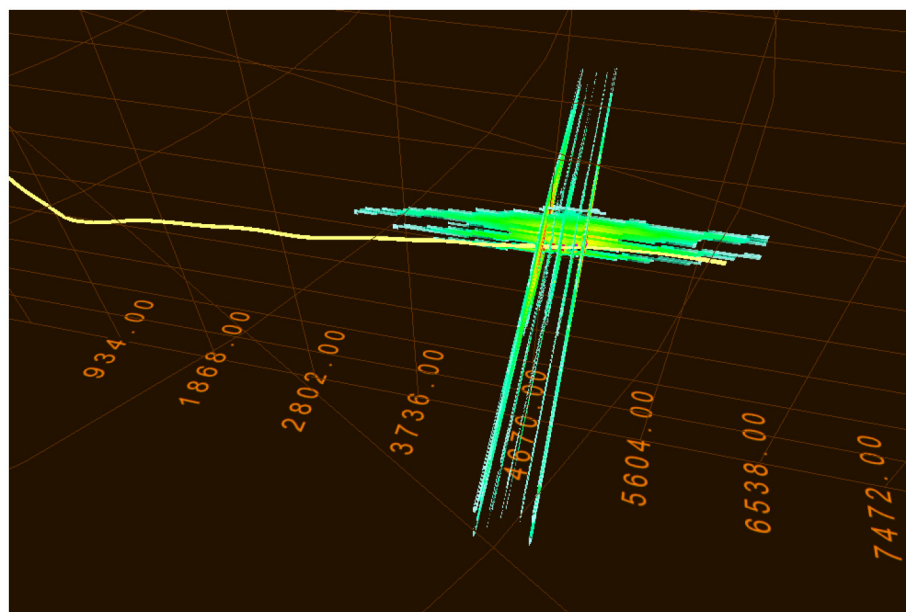


Figure 8—Transverse and longitudinal fractures initiating from a treatment stage consisting of five perforation cluster. The wellbore is shown in yellow, and the fractures are shown with final proppant concentration (green color is approximately 0.5 lb/ft³). The five transverse fractures, initiating from the five perforation cluster, can be seen growing into and out of the page. The longitudinal fracture runs with the wellbore for over 2000 ft.

of the rock mass around the propagating longitudinal fracture. Under these conditions the perforation pressure drop has no effect on diversion or control of fracture spacing or initiation. Many treatments are currently conducted using perforation cluster spacing of less than 50 feet. A longitudinal fracture can easily extend over this distance and interconnect all perforation clusters in a stage within seconds of breakdown.

Incidentally, flow of slurry through this restricted cement-formation annulus, longitudinal fracture, and transverse fractures offset to the perforation entry points is a likely cause for many of the observed rapid screenouts in cemented liner completions. Shear failure of the borehole wall, instead of simple Mode I tensile failure, is also a contributor to high tortuosity pressure drops and early screenouts, even with uncemented liner completions.

When growth of the longitudinal fracture exceeds stage-spacing, complete insulation of a treatment to a given interval of the wellbore will be virtually impossible, no matter what diversion technique (cemented or uncemented) is used inside the well itself. Such behavior might explain situations such as those shown in Fig. 9, where fracture growth, indicated by radioactive tracer systems, has bypassed internal uncemented wellbore isolation packers. Incidentally, the stress exerted by the packer element on the borehole wall conforms exactly to the assumed conditions of a non-penetrating fluid that have been used in the development of the wellbore breakdown equations (Roundtree, Barree, and Eberhard, 2009). In this case there is no hydraulic fluid pressure to invade the induced longitudinal fracture and extend shear planes into transverse hydraulic fractures. The dominant fracture plane will be longitudinal when initiated by a packer-setting event. The location of the packer element may represent the lowest tangential stress point along the well. The high concentration of tracer at the middle packer in Fig. 9 may be indicative of this mechanism.

The longitudinal growth components can also impact stage-to-stage or cluster-to-cluster stress-shadowing. In such cases, the compressive stress generated by the transverse fractures perpendicular to each other is not the only consideration, and the impacts of the longitudinal fracture, that may have penetrated an as of yet untreated stage, need to be investigated. For instance, the far right stages in Fig. 9 (yellow) will obviously have impacts on the initiation and fracture propagation of the other two

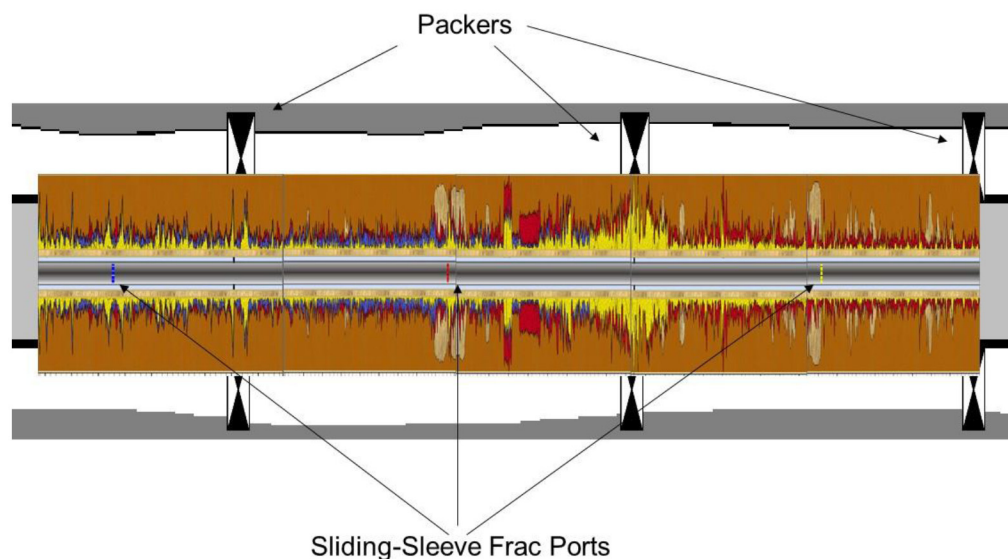


Figure 9—Three treatment stages of a wellbore using a sliding-sleeve fracture initiation system and tagged with radioactive tracer systems. The stage to the right of the figure is indicated by a yellow frac port and is associated with the yellow tracer. The stage in the center is indicated by a red frac port and is associated with the red tracer. The stage to the left is indicated by a blue frac port and is associated with the blue tracer. The tracers indicate all three stages bypass at least one of the intended diversion points.

displayed stages. The existence of the longitudinal fracture at the time a subsequent port opens provides a possible path of low resistance for fluid entry. This may, depending on rock fabric and frequency of existing rock fissures, preclude generation of any new transverse fractures.

A final consideration from the treatment design standpoint is the impacts these near-wellbore complexities have on proppants and near-wellbore conductivity. When compared to vertical wells where the hydraulic fracture may intersect a significant portion of the wellbore, with maximum fracture aperture at the well, horizontal wells have a significantly reduced entry profile for fluids flowing from the reservoir into the wellbore. Convergent flow at the wellbore results in increased non-Darcy flow, and the location of the wellbore relative to the main fracture planes can result in negative gravity and capillary effects. The fracture complexity of the near-wellbore region, which can result in shear-induced components and extremely narrow widths, can have a negative impact on fracture conductivity in the area where flow velocity is maximized and conductivity is most critical.

The geometry of the fracture connection to the wellbore may dominate treatment efficiency and may result in poor stimulation and missed reserves. It is disturbing that the local near-well conditions of borehole failure and fracture geometry, which may control the success or failure of job placement and production, are so often ignored in well and hydraulic fracture design procedures. The industry appears to have become focused on far-field fracture geometry, well spacing, well pattern geometry, and generation of fracture system “complexity” or “SRV” while forgetting or ignoring the fact that the fracture system must connect efficiently to the well to provide effective stimulation. Fluid velocity and potential gradient in the far-field (more than 100 feet from the well) are infinitesimal and fracture conductivity is less critical. Near the well, where potential gradient and velocity are high, fracture conductivity is critical. This conductivity is generated through a combination of fracture aperture, continuity or tortuosity, proppant conductivity, multiphase and non-Darcy flow conditions (including water cut), and reservoir fluid PVT properties. Details of the borehole failure mechanisms and local fracture morphology cannot be ignored and deserve much more in-depth study than they have been afforded to date.

Conclusions

This paper provides the mathematical development of modified Kirsch breakdown equations for horizontal and deviated wellbores. The inclination angle of the well, along with its azimuth orientation relative to the maximum horizontal plane, impacts the magnitude of the breakdown pressure. In addition to this magnitude, the paper discusses the orientation and initiation planes of the fractures as they exit the wellbore and the effects such tortuosity can have. The following conclusions are offered:

1. When considering the calculation of breakdown pressures and the orientation of hydraulic fractures initiated from a wellbore, the effects of highly laminated systems, such as those that occur in most unconventional reservoirs, are frequently ignored. However, such systems can and will have an impact on fracture initiation, leading to shear failure conditions and the potential for extremely complex near-wellbore fracture conditions.
2. Under many fracture initiation conditions in deviated wells, a longitudinal component of a fracture will occur that runs parallel with the wellbore. These longitudinal fractures may be short or run along a significant portion of the wellbore, however, they are almost impossible to eliminate completely. Such behavior implies that the commonly viewed picture of a horizontal wellbore with multiple transverse or oblique fractures initiating from the wellbore at controlled entry points is too simplistic.
3. Due to certain wellbore orientation conditions, as they relate to in-situ stress conditions, it is not unusual for required breakdown pressure gradients to exceed the overburden gradient by up to 40%. Such situations can lead to the inability to achieve breakdown or to exceeding tubular and wellhead pressure limits. In wells that have experienced breakdown issues, a review of such considerations may provide some explanations.
4. Breakdown conditions can have significant impacts on treatment design and analysis. Perforation breakdown may be hampered in certain orientations. Longitudinal fracture components, which will likely occur at some level, may provide communication between stages no matter what type of wellbore diversion is used. The generated near-wellbore tortuosity can cause screenouts and impact conductivity, which will lead to the loss of treated reservoir and reserves.

Nomenclature

D_{tv}	= true vertical depth, L, ft
E	= Young's modulus, m/Lt^2 , psi
P_o	= far-field pore pressure at true vertical depth, m/Lt^2 , psi
P_{ob}	= overburden pressure, m/Lt^2 , psi
P_w	= wellbore fluid pressure, m/Lt^2 , psi
r	= distance from wellbore, L, in
r_w	= wellbore radius, L, in
S_x	= transformed x-direction stress, m/Lt^2 , psi
S_{xy}	= shear stress in x-y plane, m/Lt^2 , psi
S_y	= transformed y-direction stress, m/Lt^2 , psi
S_{yz}	= shear stress in y-z plane, m/Lt^2 , psi
S_z	= transformed z-direction stress (along axis of wellbore), m/Lt^2 , psi
S_{zx}	= shear stress in z-x plane, m/Lt^2 , psi
α	= wellbore inclination angle, degrees
α_v	= Biot's coefficient in the vertical plane, unitless
β	= azimuth relative to the direction of maximum horizontal stress, degrees
γ_{ob}	= overburden pressure gradient, m/L^2t^2 , psi/ft

γ_p	= pore pressure gradient, m/L^2t^2 , psi/ft
ϵ_x	= regional tectonic strain, $\Delta L/L$, microstrains
θ	= angle from direction of minimum in-situ stress, degrees
λ	= the angle around the circumference of the well, degrees
ν	Poisson's ratio, $\Delta L/L/\Delta L/L$, unitless
σ_h	= minimum horizontal in-situ stress, m/Lt^2 , psi
σ_H	= maximum horizontal in-situ stress, m/Lt^2 , psi
σ_r	= radial stress, m/Lt^2 , psi
σ_t	= tangential (hoop) stress, m/Lt^2 , psi
σ_{tect}	= regional tectonic stress, m/Lt^2 , psi
σ_v	= vertical net (intergranular) stress, m/Lt^2 , psi
σ_z	= axial well stress, m/Lt^2 , psi
$\tau_{r\theta}$	= shear stress in the plane of the borehole stress, m/Lt^2 , psi

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