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Inflow Performance and Production Forecasting of Horizontal Wells With Multiple Hydraulic Fractures in Low-Permeability Gas Reservoirs

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ABSTRACT

The applications of horizontal well technology coupled with hydraulic fracturing in the development of low permeability gas reservoirs are investigated.

Analytical single phase and multiphase pseudo-steady state inflow performance equations for a horizontal wellbore intersecting multiple hydraulic fractures are presented. The effects of permeability anisotropy, wellbore location eccentricity, and fracture characteristics are considered. The phenomena of non-Darcy flow is also accounted for in the inflow equation associated with the flow of gas phase. These solutions provide a basis for the productivity evaluation and production forecasting of horizontal wells with multiple hydraulic fractures.

A new analytical method for predicting the future production performance of gas reservoirs is derived. The method is based on material balance analyses in which explicit pressure expressions as functions of cumulative production data and gas properties are derived for various types of gas reservoirs. Detailed production forecasting procedures are developed by using the derived pressure functions and appropriate inflow performance equations. The new production forecasting method has been further implemented on a computer. As a result, the future production performance of a gas reservoir can be easily predicted for any given scheme of horizontal drilling coupled with hydraulic fracturing, and combinations of reservoir rock and fluid properties.

With an economic evaluation, the new method may be used in the initial screening of horizontal drilling and hydraulic fracturing prospects in low permeability gas reservoirs. It may also be used in daily production operations and reservoir management. A hypothetical case study is presented which demonstrates the applicability of the new inflow performance equations and production forecasting method in the recovery optimization of a low permeability gas reservoir by combining horizontal drilling and hydraulic fracturing technology.

References and Illustrations at end of paper.

INTRODUCTION

As an environmentally sound alternative to crude oil, natural gas is becoming an increasingly significant energy resource. Many low permeability gas reservoirs are historically considered to be non-commercial due to low production rates. Most vertical wells drilled in tight gas reservoirs are stimulated using hydraulic fracturing and/or acidizing treatments to attain economical flow rates.

Recently, studies show that horizontal well technology may provide another viable approach for tapping low permeability gas reservoirs[1-4]. The combination of horizontal well drilling and hydraulic fracturing appears even more attractive[5,6]. However, the success of producing low permeability gas reservoirs by coupling horizontal well technology and hydraulic fracturing relies on careful reservoir and production performance analyses, which require the use of appropriate inflow performance equations and an adequate production forecasting method for analyzing the performance of horizontal wells with multiple hydraulic fractures.

Various steady state and pseudo steady state inflow performance equations for horizontal wells in non-fractured reservoirs have been reported in the literature[7-15]. The effects of natural and induced fractures on horizontal well productivity have been numerically or analytically investigated by various authors[5,9,10]. More recently, analytical inflow performance equations for a horizontal well intersecting discrete natural fractures have also been presented[16].

In this study, analytical inflow performance equations for a horizontal well with multiple hydraulic fractures are derived. A new semi-analytical production forecasting method for predicting production performance of depletion type and natural water drive gas reservoirs are developed based on material balance analyses. The applicability of the derived inflow performance equations and production forecasting methods in exploiting low permeability gas reservoirs using a combination of horizontal well technology and hydraulic fracturing are demonstrated by a case study.

INFLOW PERFORMANCE EQUATIONS FOR A HORIZONTAL WELL WITH MULTIPLE HYDRAULIC FRACTURES

Single Phase Inflow Performance Equations:

Comprehensive pseudo-steady state inflow performance equations for a horizontal wellbore intersecting a number of uniformly spaced, parallel natural fractures have been presented in the literature[16]. The single liquid phase inflow equation for a fully penetrating horizontal well intersecting natural fractures, as shown in Figure 1, is given as:

$$q = \frac{2\pi k_h h(\bar{P} - P_{wf})}{\mu B}\left\{\frac{2bC_1}{\beta h \ln\left[\left(\frac{4ah}{\gamma C_{A1} r_w^2}\right)\left(\frac{4\beta}{(1+\beta)^2}\right)\right]} + \frac{NC_2}{\frac{1}{3}arccosh\left[\frac{\cosh\left(\frac{\pi b \sin^2\alpha}{2aN}\right)}{\sin\left(\frac{\pi L_f \sin\alpha}{2a}\right)}\right] + \frac{\beta h}{L}\ln\left(\frac{h}{h_f}\right) + \frac{k_h h}{2k_f c}\ln\left(\frac{4h_f L_f \sin\alpha}{\gamma C_{A2} r_w^2}\right)}\right\} \qquad (1)$$

The single phase inflow equation for a non-fully penetrating well intersecting natural fractures, as shown in Figure 2, is given as:

$$q = \frac{2\pi k_h h(\bar{P} - P_{wf})}{\mu B}\left\{\frac{C_1}{\frac{1}{3}arccosh\left[\frac{\cosh\left(\frac{\pi a}{2b}\right)}{\sin\left(\frac{\pi L}{2b}\right)}\right] + \frac{\beta h}{L}\ln\left(\frac{h}{2\pi r_w \cos\left(\frac{\pi\theta}{h}\right)}\frac{2\beta}{1+\beta}\right)} + \frac{2C_2}{\left[\ln\left(\frac{r_e'}{r_w}\right) - A' + S_{CA} + S\right] + \frac{\beta h}{L}\ln\left(\frac{h}{h_f}\right) + \frac{k_h h}{2k_f c}\ln\left(\frac{4h_f L_f \sin\alpha}{\gamma C_{A2} r_w^2}\right)} + \frac{(N-2)C_2}{\frac{1}{3}arccosh\left[\frac{\cosh\left(\frac{\pi L \sin^2\alpha}{2aN}\right)}{\sin\left(\frac{\pi L_f \sin\alpha}{2a}\right)}\right] + \frac{\beta h}{L}\ln\left(\frac{h}{h_f}\right) + \frac{k_h h}{2k_f c}\ln\left(\frac{4h_f L_f \sin\alpha}{\gamma C_{A2} r_w^2}\right)}\right\} \qquad (2)$$

$$\text{where } r_e' = \frac{\sqrt{\frac{a}{2}\left[b - \frac{(N-2)L}{N}\right]}}{\pi} \qquad (3)$$

$$S_{CA} = \ln\sqrt{\frac{30.8828}{C_{AF}}} \qquad (4)$$

$$S = -\ln\left(\frac{L_f}{4r_w}\right) \qquad (5)$$

$$A' = 0.738$$

In equations (1) and (2), C_1 and C_2 are calculated by the following equations assuming an open hole completion:

$$C_1 = \frac{1}{1+\xi} \qquad (6)$$

$$C_2 = \frac{\xi}{1+\xi} \qquad (7)$$

$$\xi = \frac{2N\beta h_f L_f}{\pi[1.5(1+\beta) - \sqrt{\beta}]r_w L} \qquad (8)$$

To induce multiple hydraulic fractures along a horizontal wellbore, the horizontal section of the well must be cased. The basic theory of rock mechanics shows that the induced multiple hydraulic fractures should be parallel to each other, and can be orthogonal to the horizontal wellbore depending on its inclination with the in-situ principal stress directions. The geometric configuration of a horizontal well with multiple hydraulic fractures is, therefore, very similar to that of a horizontal well intersecting natural fractures as shown in Figure 1 and Figure 2.

If the horizontal wellbore is completely perforated, equation (1) and equation (2) can be used directly to model the inflow performance characteristics of horizontal wells with multiple hydraulic fractures. In most cases, however, hydraulic fractures are induced primarily due to very low rock matrix permeability, and subsequent negligible direct flow from the rock matrix to the wellbore. The horizontal well is therefore only perforated at the fracture intervals, and the production is primarily via the hydraulic fractures. The two coefficients, C_1 and C_2, become $C_1 = 0$ and $C_2 = 1.0$. Substituting them into equation (1) and equation (2), one obtains the inflow performance equations for a horizontal well with multiple hydraulic fractures when the wellbore are cased and selectively perforated at the fractures. Hence, the inflow equation for a fully penetrating horizontal well with multiple hydraulic fractures is given as:

$$q = \frac{2\pi k_h h(\bar{P} - P_{wf})}{\mu B}\left\{\frac{N}{\frac{1}{3}\operatorname{arccosh}\left[\frac{\cosh\left(\frac{\pi b \sin^2\alpha}{2aN}\right)}{\sin\left(\frac{\pi L_f \sin\alpha}{2a}\right)}\right] + \frac{\beta h}{L}\ln\left(\frac{h}{h_f}\right) + \frac{k_h h}{2k_f c}\ln\left(\frac{4h_f L_f \sin\alpha}{\gamma C_{A2} r_w^2}\right)}\right\} \quad \text{............ (9)}$$

The inflow equation for a non fully penetrating horizontal well with multiple hydraulic fractures is given as:

$$q = \frac{2\pi k_h h(\bar{P} - P_{wf})}{\mu B}\left\{\frac{2}{\left[\ln\left(\frac{r_e'}{r_w}\right) - A' + S_{CA} + S\right] + \frac{\beta h}{L}\ln\left(\frac{h}{h_f}\right) + \frac{k_h h}{2k_f c}\ln\left(\frac{4h_f L_f \sin\alpha}{\gamma C_{A2} r_w^2}\right)} + \frac{(N-2)}{\frac{1}{3}\operatorname{arccosh}\left[\frac{\cosh\left(\frac{\pi L \sin^2\alpha}{2aN}\right)}{\sin\left(\frac{\pi L_f \sin\alpha}{2a}\right)}\right] + \frac{\beta h}{L}\ln\left(\frac{h}{h_f}\right) + \frac{k_h h}{2k_f c}\ln\left(\frac{4h_f L_f \sin\alpha}{\gamma C_{A2} r_w^2}\right)}\right\} \quad \text{............ (10)}$$

It is generally assumed that hydraulically induced fractures will fully penetrate the whole reservoir thickness. Substituting $h = h_f$ into equation (9) and equation (10), one obtains equation (11) and equation (12) respectively.

$$q = \frac{2\pi k_h h(\bar{P} - P_{wf})}{\mu B}\left\{\frac{N}{\frac{1}{3}\operatorname{arccosh}\left[\frac{\cosh\left(\frac{\pi b \sin^2\alpha}{2aN}\right)}{\sin\left(\frac{\pi L_f \sin\alpha}{2a}\right)}\right] + \frac{k_h h}{2k_f c}\ln\left(\frac{4hL_f \sin\alpha}{\gamma C_{A2} r_w^2}\right)}\right\} \quad \text{............ (11)}$$

$$q = \frac{2\pi k_h h(\bar{P} - P_{wf})}{\mu B}\left\{\frac{2}{\left[\ln\left(\frac{r_e'}{r_w}\right) - A' + S_{CA} + S\right] + \frac{k_h h}{2k_f c}\ln\left(\frac{4hL_f \sin\alpha}{\gamma C_{A2} r_w^2}\right)} + \frac{(N-2)}{\frac{1}{3}\operatorname{arccosh}\left[\frac{\cosh\left(\frac{\pi L \sin^2\alpha}{2aN}\right)}{\sin\left(\frac{\pi L_f \sin\alpha}{2a}\right)}\right] + \frac{k_h h}{2k_f c}\ln\left(\frac{4hL_f \sin\alpha}{\gamma C_{A2} r_w^2}\right)}\right\} \quad \text{............ (12)}$$

If the hydraulically induced fractures are orthogonal to the wellbore, the inflow performance equations can be further simplified by substituting $\alpha = 90°$ into equations (11) and (12) as:

$$q = \frac{2\pi k_h h(\bar{P} - P_{wf})}{\mu B}\left\{\frac{N}{\frac{1}{3}\operatorname{arccosh}\left[\frac{\cosh\left(\frac{\pi b}{2aN}\right)}{\sin\left(\frac{\pi L_f}{2a}\right)}\right] + \frac{k_h h}{2k_f c}\ln\left(\frac{4hL_f}{\gamma C_{A2} r_w^2}\right)}\right\} \quad \text{............ (13)}$$

$$q = \frac{2\pi k_h h(\bar{P} - P_{wf})}{\mu B}\left\{\frac{2}{\left[\ln\left(\frac{r_e'}{r_w}\right) - A' + S_{CA} + S\right] + \frac{k_h h}{2k_f c}\ln\left(\frac{4hL_f}{\gamma C_{A2} r_w^2}\right)} + \frac{(N-2)}{\frac{1}{3}\operatorname{arccosh}\left[\frac{\cosh\left(\frac{\pi L}{2aN}\right)}{\sin\left(\frac{\pi L_f}{2a}\right)}\right] + \frac{k_h h}{2k_f c}\ln\left(\frac{4hL_f}{\gamma C_{A2} r_w^2}\right)}\right\} \quad \text{............ (14)}$$

Equation (9) and Equation (10) may be considered as the general single liquid phase inflow performance equations for horizontal wells with multiple hydraulic fractures.

Multiphase Inflow Performance Equations:

Once the single phase inflow performance equations are obtained, the multiphase inflow equations can be derived by introducing the reservoir properties, such as relative permeability and capillary pressure, into the corresponding single phase equations. Using equation (9), the multiphase inflow performance equations for a fully penetrating horizontal well with multiple hydraulic fractures can be obtained as follows:

$$q_o = \frac{7.082 * 10^{-3} k_{rom} k_h h(\bar{P}_o - P_{wf})}{\mu_o B_o} \times \left\{\frac{N}{\frac{1}{3}\operatorname{arccosh}\left[\frac{\cosh\left(\frac{\pi b \sin^2\alpha}{2aN}\right)}{\sin\left(\frac{\pi L_f \sin\alpha}{2a}\right)}\right] + \frac{\beta h}{L}\ln\left(\frac{h}{h_f}\right) + \frac{k_{rom} k_h h}{2k_{rof} k_f c}\ln\left(\frac{4h_f L_f \sin\alpha}{\gamma C_{A2} r_w^2}\right)}\right\} \quad \text{............ (15)}$$

$$q_w = \frac{7.082 * 10^{-3} k_{rwm} k_h h(\bar{P}_w - P_{wf})}{\mu_w B_w} \times \left\{\frac{N}{\frac{1}{3}\operatorname{arccosh}\left[\frac{\cosh\left(\frac{\pi b \sin^2\alpha}{2aN}\right)}{\sin\left(\frac{\pi L_f \sin\alpha}{2a}\right)}\right] + \frac{\beta h}{L}\ln\left(\frac{h}{h_f}\right) + \frac{k_{rwm} k_h h}{2k_{rwf} k_f c}\ln\left(\frac{4h_f L_f \sin\alpha}{\gamma C_{A2} r_w^2}\right)}\right\} \quad \text{............ (16)}$$

$$q_g = \frac{k_{rgw} k_h h [m(\bar{P}_g) - m(P_{wf})]}{1422\,T} \times \left\{ \frac{N}{\frac{1}{3}\operatorname{arccosh}\left[\dfrac{\cosh\left(\frac{\pi b \sin^2\alpha}{2aN}\right)}{\sin\left(\frac{\pi L_f \sin\alpha}{2a}\right)}\right] + \frac{\beta h}{L}\ln\left(\frac{h}{h_f}\right) + \frac{k_{rgw} k_h h}{2 k_{rgf} k_f C}\ln\left(\frac{4 h_f L_f \sin\alpha}{\gamma C_{A2} r_w^2}\right)} \right\} \qquad (17)$$

The equation for gas flow can also be expressed in terms of pressure squared as:

$$q_g = \frac{k_{rgw} k_h h (\bar{P}_g^2 - P_{wf}^2)}{1422 \mu_g Z_g T} \times \left\{ \frac{N}{\frac{1}{3}\operatorname{arccosh}\left[\dfrac{\cosh\left(\frac{\pi b \sin^2\alpha}{2aN}\right)}{\sin\left(\frac{\pi L_f \sin\alpha}{2a}\right)}\right] + \frac{\beta h}{L}\ln\left(\frac{h}{h_f}\right) + \frac{k_{rgw} k_h h}{2 k_{rgf} k_f C}\ln\left(\frac{4 h_f L_f \sin\alpha}{\gamma C_{A2} r_w^2}\right)} \right\} \qquad (18)$$

and

$$P_{cow} = \bar{P}_o - \bar{P}_w \qquad (19)$$

$$P_{cog} = \bar{P}_g - \bar{P}_o \qquad (20)$$

Similarly, the multiphase inflow performance equations for a non-fully penetrating horizontal wells with multiple hydraulic fractures can be obtained by introducing relative permeability and capillary pressure terms into equation (10).

Effect of Non-Darcy Flow on Gas Inflow Equations:

In the above discussions, it is assumed that the effect of non-Darcy flow is negligible. However, in the vicinity of and within a horizontal wellbore intersecting natural fractures, flow velocity may be high enough that non-Darcy flow effects may become significant and can not be neglected. This is particularly true for gas production.

Assume the fracture length is small compared to the dimensions of its sub-drainage block so that pseudo-radial flow exists within the sub block. The effect of non-Darcy flow can then be incorporated by introducing approximate non-Darcy coefficients into equations (17) for fully penetrating horizontal wells with multiple hydraulic fractures as[17,18]:

$$q_g = \frac{k_{rgw} k_h h [m(\bar{P}_g) - m(P_{wf})]}{1422\,T} \times \left\{ \frac{N}{\frac{1}{3}\operatorname{arccosh}\left[\dfrac{\cosh\left(\frac{\pi b \sin^2\alpha}{2aN}\right)}{\sin\left(\frac{\pi L_f \sin\alpha}{2a}\right)}\right] + \frac{\beta h}{L}\ln\left(\frac{h}{h_f}\right) + D_1\overline{q_{gf}} + \frac{k_{rgw} k_h h}{2 k_{rgf} k_f C}\left[\ln\left(\frac{4 h_f L_f \sin\alpha}{\gamma C_{A2} r_w^2}\right) + 2D_2\overline{q_{gf}}\right]} \right\} \qquad (21)$$

Similarly, one can show that the gas inflow performance equation with the non-Darcy flow effects for a non-fully penetrating horizontal well intersecting hydraulic fractures can be obtained as:

$$q_g = \frac{k_{rgw} k_h h [m(\bar{P}) - m(P_{wf})]}{1422T} \times \left\{ \frac{2}{\left[\ln\left(\frac{r_e'}{r_w}\right) - A' + S_{CA} + S\right] + \frac{\beta h}{L}\ln\left(\frac{h}{h_f}\right) + D_1\overline{q_{gf}} + \frac{k_h h}{2 k_{hf} C}\left[\ln\left(\frac{4 h_f L_f \sin\alpha}{\gamma C_{A2} r_w^2}\right) + 2D_2\overline{q_{gf}}\right]} + \frac{(N-2)}{\frac{1}{3}\operatorname{arccosh}\left[\dfrac{\cosh\left(\frac{\pi L \sin^2\alpha}{2aN}\right)}{\sin\left(\frac{\pi L_f \sin\alpha}{2a}\right)}\right] + \frac{\beta h}{L}\ln\left(\frac{h}{h_f}\right) + D_1\overline{q_{gf}} + \frac{k_h h}{2 k_{hf} C}\left[\ln\left(\frac{4 h_f L_f \sin\alpha}{\gamma C_{A2} r_w^2}\right) + 2D_2\overline{q_{gf}}\right]} \right\} \qquad (22)$$

where $D_1 = \dfrac{F_1 k_h h}{1422T}$ .... (23)

$$F_1 = 3.61\times10^{-12}\frac{\beta_1 T \gamma_g}{\mu_{gw} h^2\left(\frac{L_f}{4}\right)} \qquad (24)$$

$$D_2 = \frac{F_2 k_{hf} C}{1422T} \qquad (25)$$

$$F_2 = 3.61\times10^{-12}\frac{\beta_2 T \gamma_g \sqrt{\sin\alpha}}{\mu_{gw} C^2 r_w} \qquad (26)$$

$$\overline{q_{gf}} = \frac{q_g}{N} \qquad (27)$$

PRODUCTION FORECASTING OF HORIZONTAL WELLS WITH MULTIPLEHYDRAULIC FRACTURES IN LOW PERMEABILITY GAS RESERVOIRS

Material Balance Analysis:

For a confined and depletion type gas reservoir, the law of conservation of mass indicates that the pressure drop should be proportional to the amount of gas withdrawn from the reservoir. Therefore, the average reservoir pressure at a given time is proportional to the corresponding cumulative gas production.

Depletion Gas Reservoirs

Assuming water influx into a gas reservoir from an adjoining aquifer is insignificant, the reservoir volume occupied by hydrocarbons will not decrease, except for a slight reduction due to connate water expansion and pore volume compaction as reservoir pressure declines during production. These types of reservoirs are termed depletion gas reservoirs. The material balance equation of such an isothermal gas reservoir is given as[17]:

$$\frac{G_p}{G} = 1 - \left[1 - \frac{(C_w S_w + C_f)\Delta P}{1 - S_{wc}}\right]\frac{E}{E_i} \qquad (28)$$

In equation (28) E is the gas expansion factor, G is the original gas in place, G_p is the cumulative gas production and $\Delta P = P_i - \bar{P}$, where P_i is the initial reservoir pressure and $\bar{P}$ is the average reservoir pressure. The second term in the bracket accounts for the hydrocarbon pore volume reduction due to connate water expansion and pore compressibility.

Generally, the gas expansion factor is approximately a linear function of reservoir pressure, and can be obtained from standard PVT analysis and hence can be approximated as:

$$E = E_i + a_e(\bar{P} - P_i) \qquad \text{............ (29)}$$

where a_e is the slope of experimental E vs. P data. Substituting equation (29) into equation (28), one can obtain the following pressure equation in quadratic form:

$$A(\bar{P} - P_i)^2 + B(\bar{P} - P_i) + C = 0 \qquad \text{............ (30)}$$

$$\text{where } A = a_e\left(\frac{C_w S_w - C_f}{1 - S_{wc}}\right) \qquad \text{............ (31)}$$

$$B = a_e + E_i\left(\frac{C_w S_w - C_f}{1 - S_{wc}}\right) \qquad \text{............ (32)}$$

$$C = E_i \frac{G_p}{G} \qquad \text{............ (33)}$$

Explicit pressure solutions as functions of production data and gas properties are obtained by solving equation (30) as follows:

$$\bar{P} = \begin{cases} P_i + \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A} & (\text{for } A \neq 0) \\ P_i - \dfrac{C}{B} & (\text{for } A = 0,\ B \neq 0) \end{cases} \qquad \text{.......... (34)}$$

In many cases, connate water expansion and pore compressibility are negligible due to the high compressibility of natural gas. As a result, the material balance equation becomes:

$$\frac{G_p}{G} = 1 - \frac{E}{E_i} \qquad \text{............ (35)}$$

Substituting equation (29) into equation (35) and after some algebraic manipulations one obtains:

$$\bar{P} = P_i - \left(\frac{E_i}{a_e}\right)\left(\frac{G_p}{G}\right) \qquad \text{............ (36)}$$

It can be shown that equation (36) can also be directly deduced from equation (34). Therefore, equation (34) will be the general pressure solutions for a depletion gas reservoir and can be used in future gas production forecasting.

Water Drive Gas Reservoirs

If the reduction in reservoir pressure leads to a significant expansion of the adjacent aquifer water and consequent influx into the reservoir, the material balance equation of such a water drive gas reservoir is[17]:

$$G_p = G - \left(\frac{G}{E_i} - W_e\right)E \qquad \text{............ (37)}$$

where W_e is the cumulative water influx resulting from the pressure drop. It is assumed that there is no difference between surface and reservoir volumes of water, and the effects of connate water expansion and pore volume compaction are negligible.

The modelling of water influx is not an easy task and involves great uncertainties since the information on the aquifer is usually minimal. This is especially true in the early stage of production planning. For simplicity, it is assumed that the aquifer is relatively small so that a pressure drop in the reservoir is instantaneously transmitted throughout the entire reservoir-aquifer system. The aquifer model is then given as[17]:

$$W_e = \bar{c}W\Delta P \qquad \text{............ (38)}$$

In equation (38), $\bar{c}$ is the total aquifer compressibility $(c_w + c_f)$, W is the total volume of water, and ΔP is the pressure drop at the original reservoir-aquifer boundary. ΔP may be approximated by $(P_i - \bar{P})$. Substituting equations (29) and (38) into equation (37), one obtains:

$$A(\bar{P} - P_i)^2 + B(\bar{P} - P_i) + C = 0 \qquad \text{............ (39)}$$

$$\text{where } A = a_e\bar{c}W \qquad \text{............ (40)}$$

$$B = \frac{a_e G}{E_i} + E_i\bar{c}W \qquad \text{............ (41)}$$

$$C = G_p \qquad \text{............ (42)}$$

Solutions of equation (39) can be obtained as:

$$\bar{P} = \begin{cases} P_i + \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A} & (\text{for } A \neq 0) \\ P_i - \dfrac{C}{B} & (\text{for } A = 0,\ B \neq 0) \end{cases} \qquad (43)$$

One can show that the second solution in equation (43) will reduce to equation (36) when the volume of the aquifer is negligible ($W = 0.0$).

When the volume of the aquifer is large, more sophisticated unsteady state aquifer models are needed to replace equation (38). It is believed that similar pressure solutions can be obtained by using the same approach.

Production Forecasting:

With derived inflow performance equations and the pressure expressions as functions of production data and reservoir fluid properties, future gas production forecasting procedures can be developed by partitioning the gas reservoir depletion process into successive time steps, during each of which the reservoir can be considered as in pseudo-steady state. In the i-th time step, the production calculations for a depletion gas reservoir can be carried out by the following equations:

$$(G_p)^i = (G_p)^{i-1} + (q_{gmax})^{i-1}\Delta t \qquad \text{............ (44)}$$

$$(RF)^i = \frac{(G_p)^i}{G} \qquad \text{............ (45)}$$

$$(A)^i = a_e\left(\frac{C_w(S_w)^{i-1} - C_f}{1 - S_{wc}}\right) \qquad \text{............ (46)}$$

$$(B)^i = a_e + E_i\left(\frac{C_w(S_w)^{i-1} - C_f}{1 - S_{wc}}\right) \qquad \text{............ (47)}$$

$$(C)^i = E_i\left(\frac{(G_p)^i}{G}\right) \qquad \text{............ (48)}$$

$$(\bar{P})^i = \begin{cases} P_i + \dfrac{-(B)^i \pm \sqrt{[(B)^i]^2 - 4(A)^i(C)^i}}{2(A)^i} & (for\ A \neq 0) \\ P_i - \dfrac{(C)^i}{(B)^i} & (for\ A = 0,\ B \neq 0) \end{cases}$$

$$\text{............ (49)}$$

$$(E)^i = E_i + a_e\left\{\left[\frac{(\bar{P})^i + P_{wf}}{2}\right] - P_i\right\} \qquad \text{............ (50)}$$

$$(Z_g)^i = 35.37\frac{((\bar{P})^i + P_{wf})}{2(E)^i T} \qquad \text{............ (51)}$$

$$(S_w)^i = \frac{S_{wc}[1 + C_w(P_i - (\bar{P})^i)]}{1 - \left(\dfrac{C_w(S_w)^{i-1} - C_f}{1 - S_{wc}}\right)[P_i - (\bar{P})^i]} \qquad \text{........... (52)}$$

$$(S_g)^i = 1 - (S_w)^i \qquad \text{............ (53)}$$

$$(J_{ghf})^i = \frac{(k_{rgm})^i k_h h}{1422(\mu_g)^i(Z_g)^i T} \times$$

$$\left\{\frac{N}{\dfrac{1}{3}arccosh\left[\dfrac{\cosh\left(\dfrac{\pi L\sin^2\alpha}{2aN}\right)}{\sin\left(\dfrac{\pi L_f\sin\alpha}{2a}\right)}\right] + \dfrac{\beta h}{L}\ln\left(\dfrac{h}{h_f}\right) + \dfrac{k_h h}{2k_f c}\ln\left(\dfrac{4h_f L_f\sin\alpha}{\gamma C_{A2} r_w^2}\right)}\right\}$$

$$\text{............ (54)}$$

$$(q_{gmax})^i = (J_{ghf})^i \times [(\bar{P}^i)^2 - (P_{wfmin})^2] \qquad \text{........... (55)}$$

The production forecasting of a depletion gas reservoir can be easily performed by implementing the above procedures on a computer.

The procedures for the production forecasting of a water drive gas reservoir are similar to those given by equations (44) through (55) except that equations (46) through (48) should be replaced by the corresponding equations for a water drive gas reservoir [equations (40) through (42)].

EXAMPLE APPLICATION

The combination of horizontal well drilling and hydraulic fracturing technologies provides a mechanism for more efficient recovery of natural gas from low permeability reservoirs. However, the success of these technologies require quantitative engineering analyses. The purpose of this case study is to show the applicability of the derived inflow performance equations and production forecasting method in the initial screening of horizontal well drilling and hydraulic fracturing schemes in exploiting a gas reservoir.

Shown in Figure 3 is the net pay isopach map of a hypothetical low permeability gas reservoir. It has an area of 736 acres, original gas in place of 9.24 tcf. Additional reservoir and well data as well as economic evaluation parameters are given in Table 1. It is proposed that horizontal drilling and hydraulic fracturing techniques be used to produce the reservoir due to low rock matrix permeability.

Depicted in Figure 4 through Figure 8 are five proposed horizontal well drilling scenarios from drilling one well to drilling five wells. It is assumed that the minimum principal in-situ stress is in the direction of NE-SW, parallel to the proposed horizontal well orientation. As a result, any induced fractures will be orthogonal to the horizontal wellbore.

Effect of Hydraulic Fractures on Horizontal Well Productivity

For the single well case shown in Figure 4, let's assume that hydraulic fractures of 400 ft are induced along the horizontal wellbore. The effect of fractures on the horizontal well productivity is shown in Figure 9, in which gas production rate is plotted as a function of producing time and number of fractures induced. One can see that at least three fractures must be created in order to obtain the same initial production capacity for a cased horizontal well compared to that of the non-stimulated open hole horizontal well. The production rate can be increased four times as much if seven fractures are created.

Effect of Additional Drilling on Reservoir Productivity

Shown in Figure 10 is the total gas production rate of the reservoir as a function of producing time and number of horizontal wells drilled, in which no fractures are induced. Consistent with common belief, the total reservoir production capacity can be dramatically increased if more wells are drilled. The total gas production rate of the reservoir is increased from 200 MSCF/D for a single well to 4300 MSCF/D for five wells.

Optimization of Horizontal Well Drilling and Hydraulic Fracturing

The fact that more drilling and additional fracturing also means increased investment indicates that there must be an optimal number of wells coupled with an optimal number of fractures which should be drilled or induced to maximize the profits and/or hydrocarbon recovery. In the following discussions, an economic evaluation is performed using gas production rate profiles generated by the derived production forecasting method in this study for different combinations of wells and fractures.

Shown in figure 11 is the discounted net present value (NPV) profile for open hole horizontal wells with no hydraulic fractures, in which the NPV of the reservoir is plotted as a function of producing time and number of wells drilled. It can be observed that the two well system depicted in Figure 5 results in the maximum profit after 20 years' production.

When hydraulic fractures are created along horizontal wellbores, it is assumed that the horizontal wells are cased and the production is solely from fractures. Figure 12 shows the NPV profile when three hydraulic fractures are created along each horizontal well. One can see that the optimal scenario in this case is to drill three wells. However, as can be seen from Figure 13 through Figure 16, the optimal drilling scenario is to drill two wells if more than three fractures are created along each well. The optimal drilling scenarios identified from Figure 11 through Figure 16 are further compared in Figure 17, in which Wx-Fy corresponding to the case of drilling x wells and creating y fractures along each well. It is evident that the optimum scheme for producing this reservoir is to drill two wells and create seven fractures on each well. Furthermore, this economically optimal scheme also maximizes the gas recovery, as can be seen from Figure 18 in which gas recovery factor is plotted as functions of producing time for the scenarios considered in Figure 17. This conclusion is only true for the assumed well length, fracture length and reservoir and well data. Different conclusion may be obtained if different sets of data are used in the analysis.

CONCLUSIONS

Analytical single phase and multiphase pseudo-steady state inflow performance equations for horizontal wells intersecting hydraulic fractures are derived. The phenomena of non-Darcy flow is considered in the gas inflow equations. These derived inflow equations provide engineers with relatively simple tools for evaluating the productivity of horizontal wells intersecting hydraulic fractures.

An analytical production forecasting method for predicting future gas production performance of a low permeability gas reservoir is derived based on a material balance analysis. The methodology can be used for initial screening of horizontal drilling and hydraulic fracturing schemes for exploiting a low permeability gas reservoir.

A case study indicates that the combination of horizontal well and hydraulic fracturing technology provides an attractive approach for exploiting low permeability gas reservoirs. It appears that the best strategy is to drill fewer horizontal wells and create more fractures. However, production forecasting and economic evaluation must be performed in order to identify the optimal schemes for horizontal well drilling and hydraulic fracturing.

NOMENCLATURE

a = reservoir width, ft.
a' = half the major axis of a drainage ellipse, ft.
A = drainage area, ft^2.
A' = constant.
b = reservoir length, parallel to horizontal well, ft.
B = formation volume factor, RB/STB.
c = fracture aperture, ft.
C_1, C_2 = weighing functions, dimensionless.
C_{A1} = shape factor corresponding to the location of a horizontal well in a vertical plane, dimensionless.
C_{A2} = shape factor corresponding to the location of a horizontal well in the plane of the vertical fracture, dimensionless.
C_{AH} = horizontal well shape factor, dimensionless.
C_{AF} = vertical fracture shape factor, dimensionless.
e = vertical distance between the axis of a horizontal well and the middle of a reservoir thickness, ft.
h = reservoir thickness, ft.
h_f = fracture height, ft.
J = productivity index, bbl/day/psi.
k = rock matrix permeability, md.
k_f = fracture permeability, md.
k_h = horizontal permeability in matrix, md.
k_v = vertical permeability in matrix, md.
k_{rom} = relative permeability of oil in rock matrix, dimensionless.
k_{rof} = relative permeability of oil in natural fractures, dimensionless.
k_{rgm} = relative permeability of gas in rock matrix, dimensionless.
k_{rgf} = relative permeability of gas in natural fractures, dimensionless.
k_{rwm} = relative permeability of water in rock matrix, dimensionless.
k_{rwf} = relative permeability of water in natural fractures, dimensionless.
L = horizontal well length, ft.
L_f = fracture length, ft.
$m(P)$ = real gas pseudo pressure, (STB psi)/(RB cp).
n = normal vector, ft.
N = number of fractures intersected by a horizontal well.
N_m = movable hydrocarbon reserve, STB.
N_p = cumulative oil production, STB.
$\bar{P}$ = average pressure, psi.
P_c = capillary pressure, psi.
P_f = pressure in a fracture, psi.
P_e = pressure at reservoir boundary, psi.
P_{wf} = bottom hole flowing pressure, psi.
q_m = flow rate directly from matrix to horizontal wellbore, STB/day.
q_f = flow rate from matrix to fractures, then from fractures to horizontal wellbore, STB/day.
q = total flow rate, STB/day.
r_w = wellbore radius, ft.
R_s = solution gas oil ratio, scf/STB.
S = skin factor, dimensionless.
S_{CA} = shape dependent pseudo-skin factor, dimensionless.
S_{gc} = critical gas saturation, dimensionless.
t = time, day.
Z_g = gas factor, dimensionless.

Greek:

α = angle between a horizontal well axis and natural fractures, degree.
β = permeability anisotropic ratio, dimensionless.
β_1 = non-Darcy flow coefficient in rock matrix, 1/ft.
β_2 = non-Darcy flow coefficient in fractures, 1/ft.
ϕ = porosity, dimensionless.
γ = 1.781, exponential of Euler's constant, dimensionless.
μ = reservoir fluid viscosity, cp.
λ = proportional constant, dimensionless.

Subscripts:

o = oil
w = water or well
g = gas
m = matrix
f = fracture
i = initial
b = bubble point

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Table 1. Reservoir, Well And Economic Data

Reservoir Porosity ϕ	0.20
Fracture Length L_f	400 ft
Fracture Aperture c	0.25 inches
Fracture Permeability k_f	4.0×10^4 md
Horizontal Well Length L	2000 ft
Horizontal Wellbore Radius r_w	6 inches
Connate Water Saturation S_{wc}	0.20
Rock Compressibility C_f	8.6×10^{-6} $psia^{-1}$
Water Compressibility C_w	3.0×10^{-6} $psia^{-1}$
Horizontal Permeability k_h	0.10 md
Vertical Permeability k_v	0.01 md
Critical Gas Saturation S_{gc}	0.05
Original Gas in Place G	9.24 tcf
Original Reservoir Pressure P_i	3300 psi
Original Gas Expansion Factor E_i	185.24 scf/rcf
Reservoir Temperature T	$200°F$
Specific Gravity γ_g	0.85
Drilling and Completion Costs	$1.5 million/well
Hydraulic Fracturing Costs	$20000/fracture

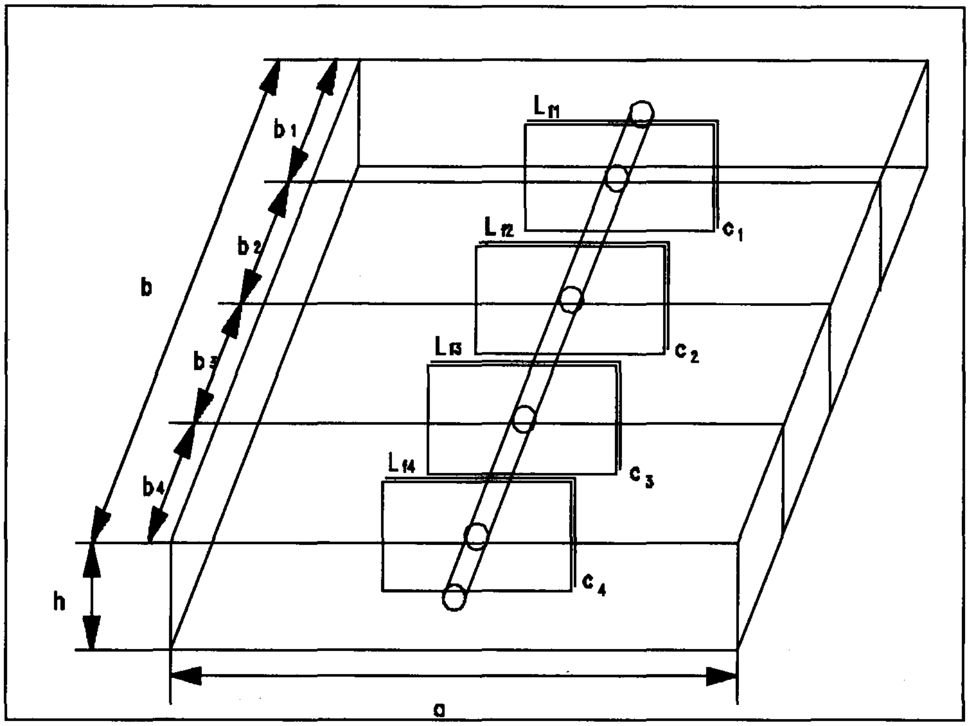

Figure **1**. A Fully Penetrating Horizontal Well Intersecting Natural Fractures

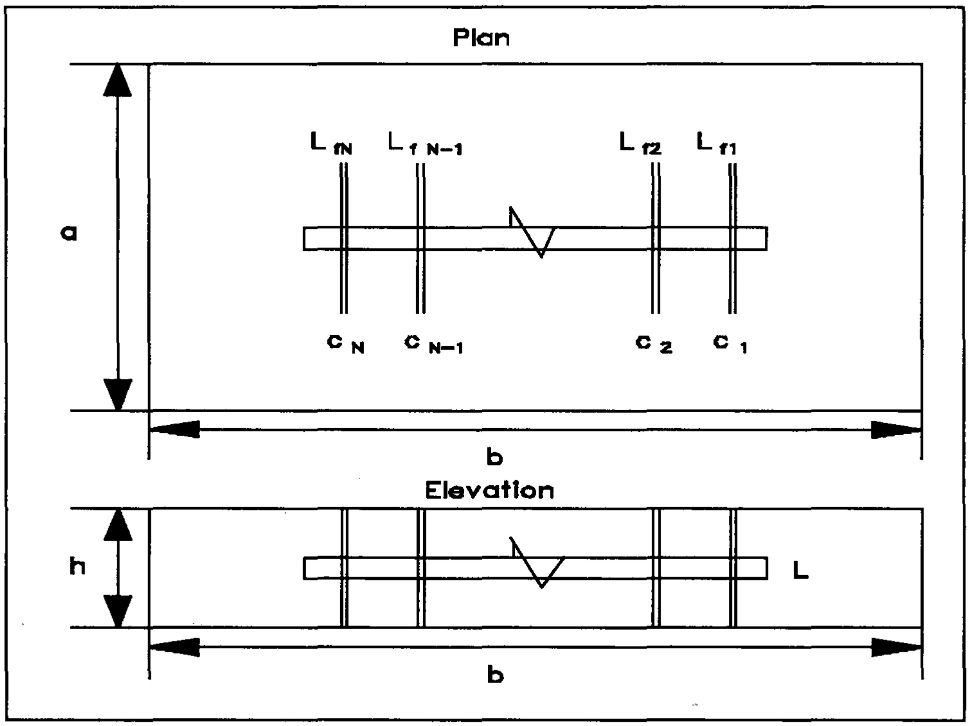

Figure **2**. A Non-Fully Penetrating Horizontal Well Intersecting Natural Fractures

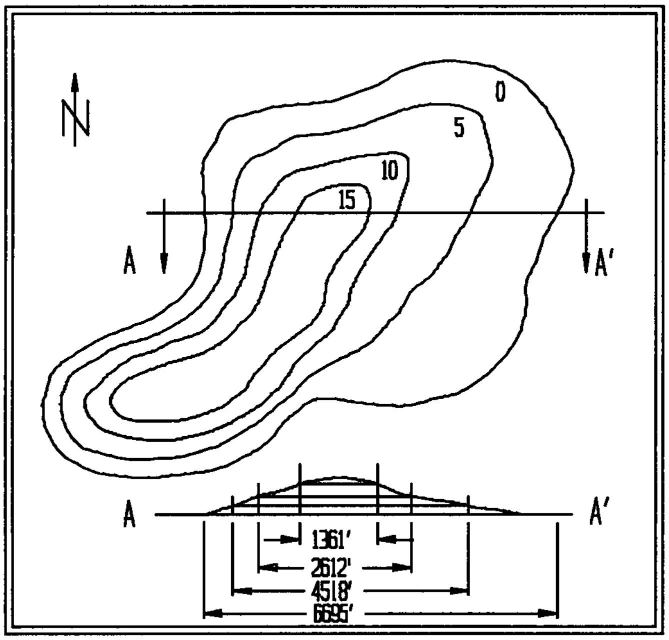

Figure 3. Net Pay Isopach Map Of The Grynberg Reservoir

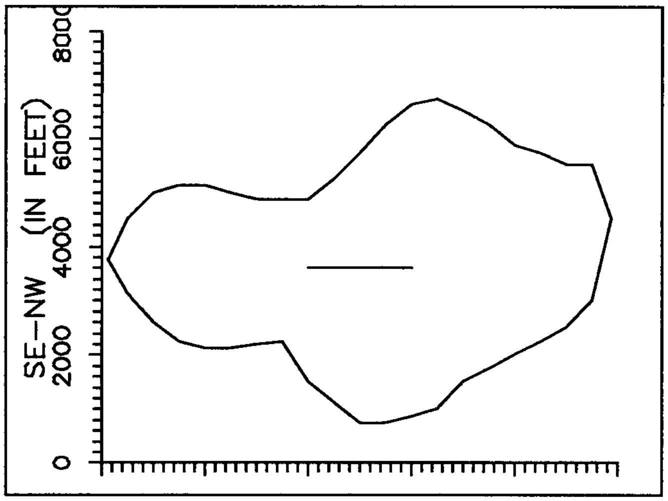

Figure 4. One Well System

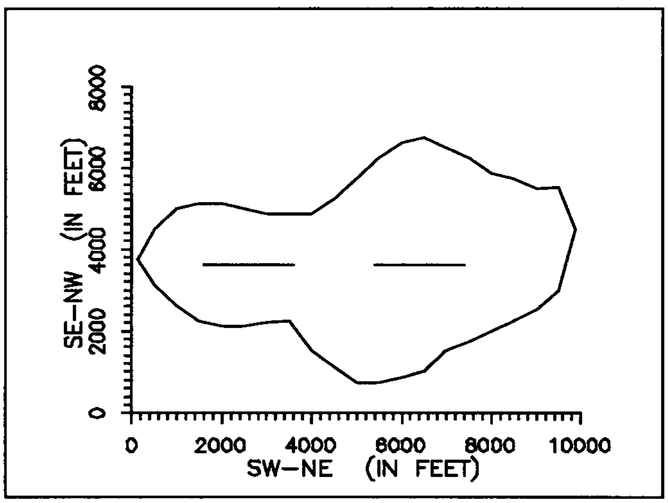

Figure 5. Two Well System

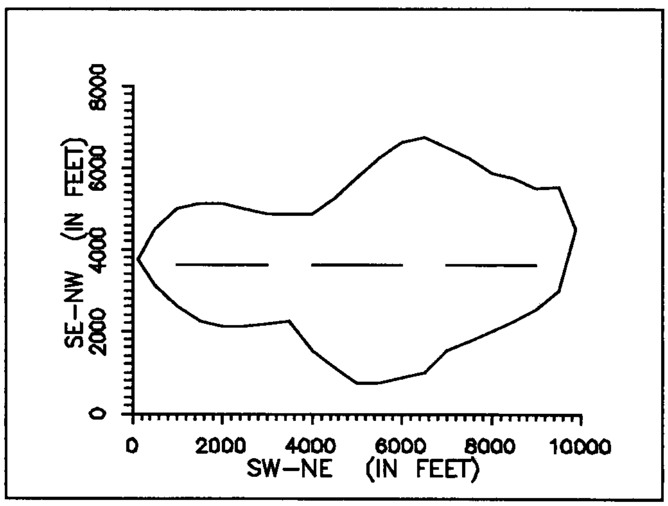

Figure 6. Three Well System

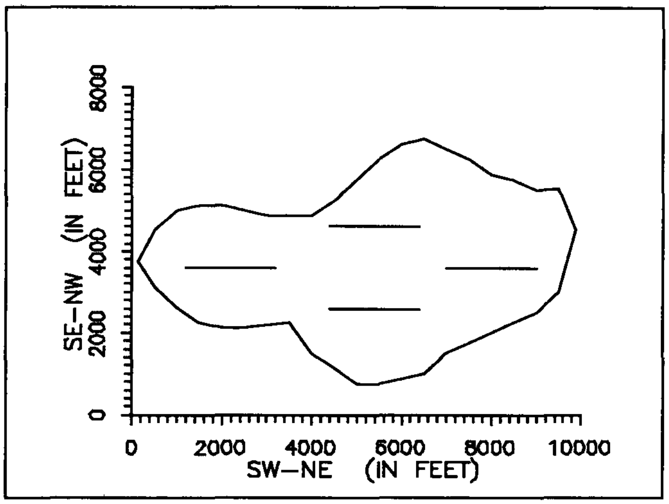

Figure 7. Four Well System

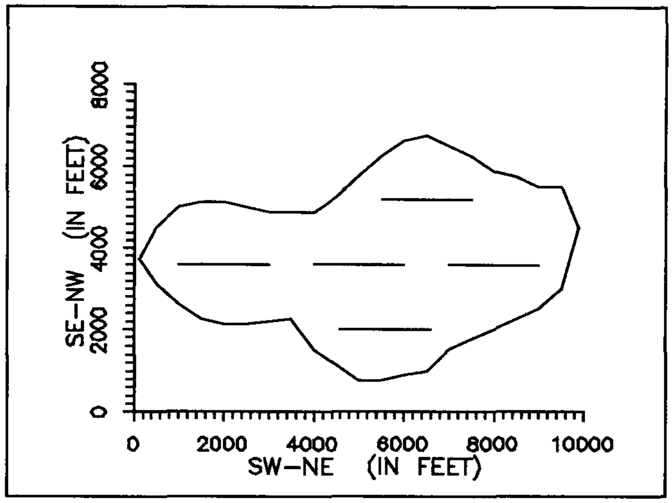

Figure 8. Five Well System

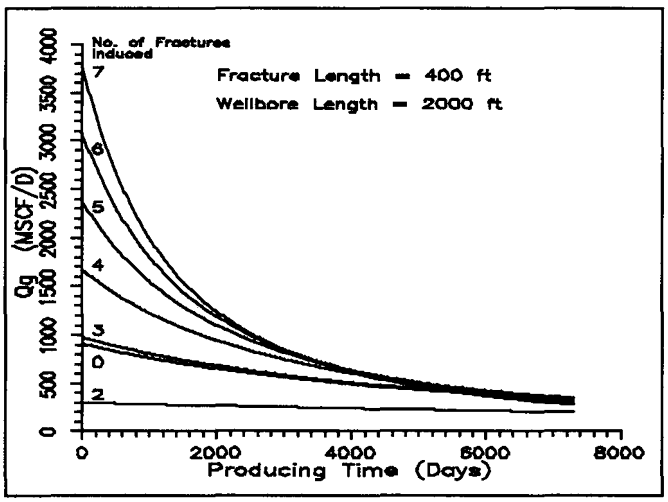

Figure 9. Gas Production Rate as a Function of Producing Time and Number of Fractures Induced

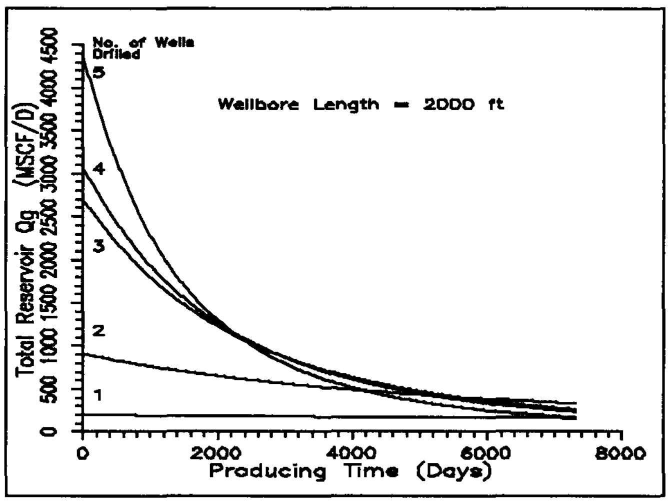

Figure 10. Total Gas Production Rate as a Function of Producing Time and Number of Wells Drilled

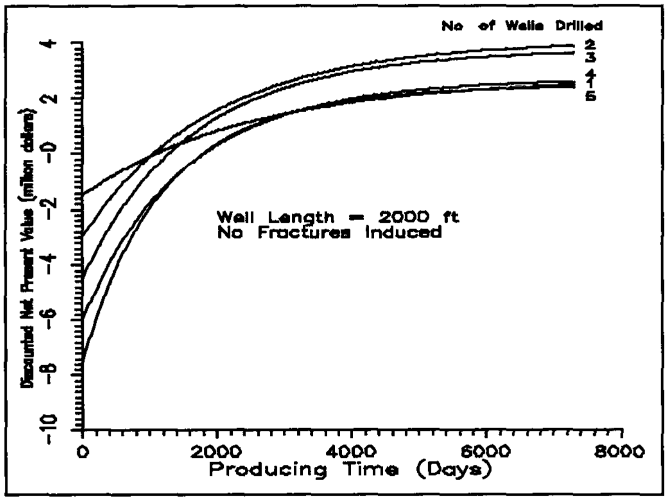

Figure 11. Discounted Net Present Value Profile for Open Hole Horizontal Wells With No Fractures

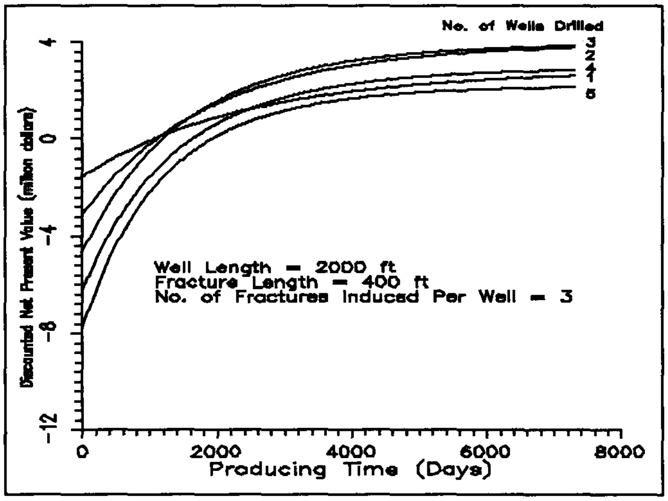

Figure 12. Discounted Net Present Value Profile for Cased Horizontal Wells With Three Fractures

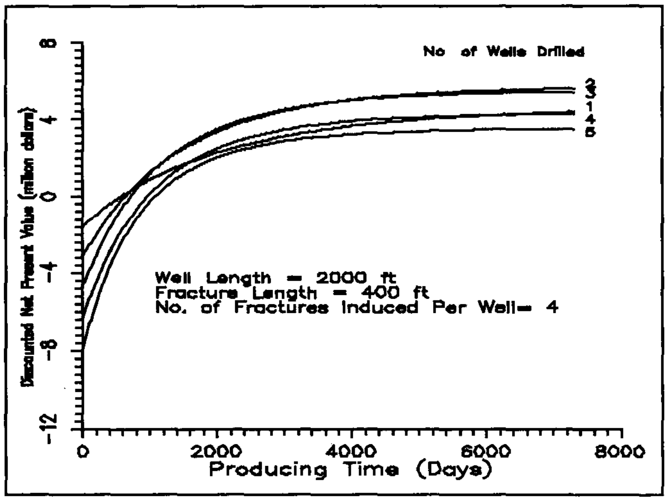

Figure 13. Discounted Net Present Value Profile for Cased Horizontal Wells With Four Fractures

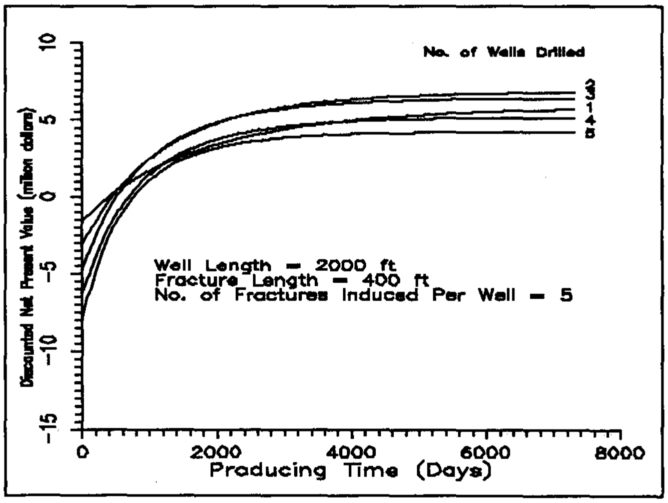

Figure 14. Discounted Net Present Value Profile for Cased Horizontal Wells With Five Fractures

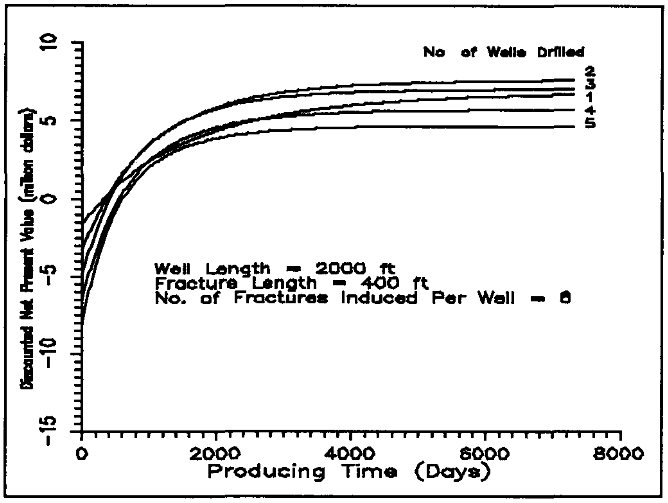

Figure 15. Discounted Net Present Value Profile for Cased Horizontal Wells With Six Fractures

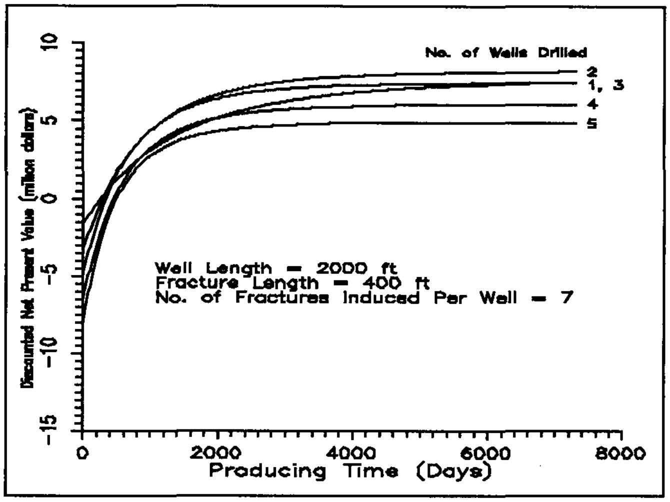

Figure 16. Discounted Net Present Value Profile for Cased Horizontal Wells With Seven Fractures

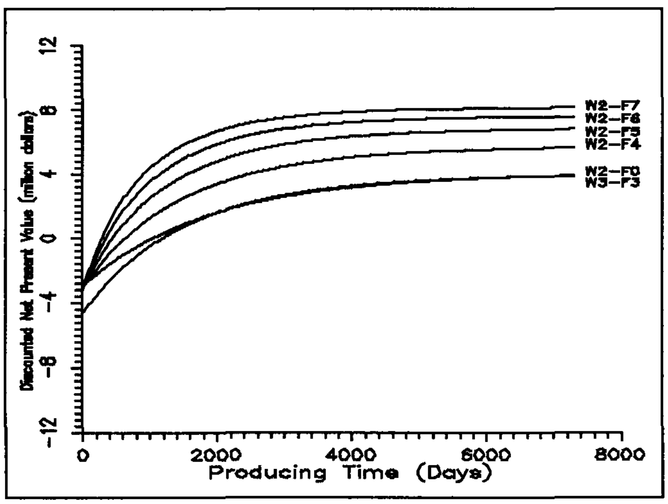

Figure 17. Comparison of the Optimal Drilling Scenarios for Different Number of Fractures Induced

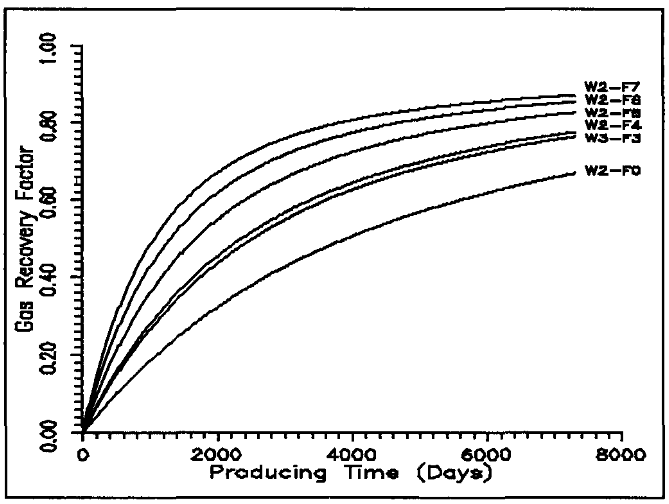

Figure 18. Gas Recovery Profile