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A Comprehensive Mechanistic Model for Two-Phase Flow in Pipelines

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ABSTRACT

A comprehensive mechanistic model has been developed for gas-liquid two-phase flow in horizontal and near horizontal pipelines. The model is able first to detect the existing flow pattern, and then to predict the flow characteristics, primarily liquid holdup and pressure drop, for the stratified, intermittent, annular, or dispersed bubble flow patterns.

A pipeline data bank has been established. The data bank includes large diameter field data culled from the A. G. A. database, and laboratory data published in the literature. Data include both black oil and compositional fluid systems.

The comprehensive mechanistic model has been evaluated against the data bank and also compared with the performance of some of the most commonly used correlations for two-phase flow in pipelines. The evaluation, based on the comparison between the predicted and the measured pressure drops, demonstrated that the overall performance of the proposed model is better than that of any of the correlations, with the least absolute average percent error and the least standard deviation.

INTRODUCTION

Prediction of flow patterns, liquid holdup and pressure loss for two-phase flow in pipelines is important for designing gas-liquid transportation systems. The traditional approach to solve the problem has been to conduct experiments and develop empirical correlations. Although these correlations have contributed significantly to the design of two-phase flow systems, they did not take into consideration the physical phenomena.

Since the mid 1970's, significant progress has been made in this area. Models have been developed to predict flow patterns. Separate Models have also been proposed for the prediction of the flow characteristics for each flow pattern, namely stratified flow, intermittent flow, annular flow and dispersed bubble flow. However, up to date, no study has been carried out to verify the consistency and the applicability of these models.

The purpose of this study is to develop a comprehensive mechanistic model for two-phase flow in pipelines by combining the most recent developments in this area. The model is then evaluated against a field and laboratory measurement data bank, and compared with several commonly used empirical correlations.

FLOW PATTERN PREDICTION MODEL

When gas and liquid flow simultaneously in a pipe, the two phases can distribute themselves in a variety of flow configurations or flow patterns, depending on operational parameters, geometrical variables as well as physical properties of the two phases. The existing flow patterns in pipelines have been classified into four major types: Stratified Flow (Stratified Smooth and Stratified Wavy), Intermittent

References and illustrations at end of paper.

Flow (Elongated Bubble Flow and Slug Flow), Annular Flow (Annular Mist Flow and Annular Wavy Flow) and Dispersed Bubble Flow. These flow patterns are shown in Fig. 1.

Flow pattern prediction is a central problem in two-phase flow analysis. The recent trend in this area is the development of mechanistic models based on the physical phenomena. The pioneering work is due to Taitel & Dukler (1976) and Taitel *et al.* (1980). Later, Barnea *et al.* (1982a, 1982b, 1985 and 1987) adopted the same approach, modified and extended the existing models to form a unified model for the entire pipe inclination angles. On the other hand, flow pattern determination, especially for the onset of slugging, has been investigated through linear stability theory by various researchers (Lin & Hanratty 1986, Andritsos 1986 and Wu *et al.* 1987). Unfortunately, this approach is mathematically complex and its solution is very involved for design purposes. Hence, the Taitel & Dukler (1976) model with some modifications is used in the present work.

Three major flow pattern transitions are identified here: The Stratified-Non Stratified transition, the Intermittent-Annular transition and the Intermittent-Dispersed Bubble transition. Stratified flow is further divided into two subregions: Stratified-Smooth and Stratified-Wavy flow.

<u>Stratified-Non Stratified Transition (S-NS):</u> The mechanism of wave growth is used for the prediction of this transition. A finite wave is assumed to exist on the gas-liquid interface of an equilibrium stratified flow. Extending the Kelvin-Helmholtz theory to analyze the stability of finite waves in pipes, Taitel & Dukler claimed that when the pressure suction force is greater than the gravity force, waves tend to grow and thus stratified flow cannot be preserved. Their analysis leads to the following criterion for this transition:

$$v_g > \left(1 - \frac{h_L}{D}\right)\left[\frac{(\rho_L - \rho_g)\, g \cos\alpha \; A_g}{\rho_g \left(\frac{dA_L}{dh_L}\right)}\right]^{1/2} \quad \text{............... (1)}$$

This transition is shown as transition A in Fig. 2 for air-water flow at atmospheric pressure in a 0.05-m diameter pipe with a inclination angle of -1°.

The stratified-slug transition is predicted satisfactorily by Eq. (1). For the stratified-annular transition, however, recent experiments conducted by Lin & Hanratty (1987) showed that the entrainment-deposition process is dominant for large diameter pipes, while for small diameter pipes wave-growth is usually the dominant mechanism. Nevertheless, no generally accepted model based on the entrainment-deposition mechanism has yet been found in the literature.

<u>Intermittent-Annular Transition (I-A):</u> When waves are unstable, the flow could change to either intermittent flow or annular flow, depending on whether there is enough liquid supply. The proposed critical liquid level was 0.5 in the Taitel & Dukler (1976) model. Barnea *et al.* (1982a) modified this criterion by taking into account possible gas void fraction in liquid slug near the transition. The revised transition is given by:

$$\frac{h_L}{D} < 0.35 \quad \text{............ (2)}$$

This is shown as transition B in Fig. 2.

<u>Intermittent-Dispersed Bubble Transition (I-DB):</u> The mechanism governing this transition is believed to be the turbulent process which breaks up bubbles and prevents bubble coalescence. Barnea *et al.* (1987) developed a unified model for the transition to dispersed bubble flow applicable to all inclination angles. For the condition considered in this study ($-15° \leq \alpha \leq 15°$) , however, the original Taitel & Dukler (1976) model is used because of its simplicity and sufficient accuracy. When the turbulent force is sufficiently high to overcome buoyant force, the gas is no longer able to stay at the top of the pipe, and dispersed bubble flow will occur. The transition criterion is expressed as:

$$v_L > \left[\frac{4\, A_g}{S_i}\,\frac{g \cos\alpha}{f_L}\left(1 - \frac{\rho_g}{\rho_L}\right)\right]^{1/2} \quad \text{............ (3)}$$

This is shown as transition C in Fig. 2.

<u>Stratified Smooth-Stratified Wavy Transition (SS-SW):</u> In stratified flow, the gas-liquid interface can be either smooth or wavy, which gives quite different results for liquid holdup and pressure drop. Waves may develop due to either the interfacial shear or as a result of instability due to the action of gravity. For waves induced by "wind" effect, Taitel & Dukler (1976) proposed the following criterion according to Jeffrey's theory:

$$v_g > \left[\frac{4\, \mu_L (\rho_L - \rho_g)\, g \cos\alpha}{s\, \rho_L\, \rho_g\, v_L}\right]^{1/2} \quad \text{............ (4)}$$

where, s is a sheltering coefficient. Values ranging from 0.01 to 0.6 have been suggested from theories and experiments in the literature. Taitel & Dukler (1976) used a value of 0.01 to match their experimental data. A recent study by Andritsos (1986) showed that the criterion given by Eq. (4) with s = 0.01 is not accurate for gas flow with liquids of high viscosity. They found that a good comparison can be obtained if a value of 0.06 is used. The sheltering coefficient may indeed be a function of liquid viscosity. In the present work the value of s = 0.06 is used. This transition is shown as transition D in Fig. 2. As can be seen, the transition line is shifted to the left of the line given by the original Taitel & Dukler model (1976).

For stratified flow in downwardly inclined pipes, waves can develop under the influence of gravity even without the presence of interfacial shear. Barnea *et al.* (1982a) presented the transition criterion as:

$$\frac{v_L}{\sqrt{g\, h_L}} > 1.5 \qquad (5)$$

In Figure 2, this transition boundary is represented by Curve E, and is terminated at the transition D where waves are agitated by interfacial shear.

INDIVIDUAL FLOW PATTERN MODELS

After predicting the actual flow pattern from the operational conditions, separate models are needed to calculate liquid holdup and pressure drop for the predicted flow pattern. These models are developed in the following section

STRATIFIED FLOW MODEL

In stratified flow, due to gravity, liquid flows in the bottom portion of the pipe while gas flows in the upper portion of the pipe, as shown in Fig. 3. Stratified flow is one of the most dominant flow patterns for two-phase flow in pipelines, particularly for flow in downwardly inclined pipes.

Over the years, various theoretical models with different degrees of complexity have been proposed for this flow pattern. Significant recent work includes the Taitel & Dukler (1976) model, the Cheremisinoff (1977) model and the Shoham & Taitel (1984) two-dimensional model. However, models considering the liquid phase velocity profile are neither easy to use nor guarantied to give better results. This is maybe one of the reason that the generalized one-dimensional two-fluid model by Taitel & Dukler (1976) is commonly used. This approach is adopted in this study.

Using the steady state one-dimensional two-fluid model approach and neglecting changes of phase velocities (or liquid level), the momentum equations for the two fluids reduce to force balances. They can be written as:

$$- A_L\left(\frac{dp}{dx}\right) + \tau_i\, S_i - \tau_{wL}\, S_L - A_L\, \rho_L\, g \sin\alpha = 0 \qquad (6)$$

$$- A_g\left(\frac{dp}{dx}\right) - \tau_i\, S_i - \tau_{wg}\, S_g - A_g\, \rho_g\, g \sin\alpha = 0 \qquad (7)$$

Under the assumptions of negligible surface tension and liquid phase hydrostatic pressure gradient, the pressure gradients in both phases are the same. Eliminating the pressure gradient from these equations results in the so called combined momentum equation:

$$\tau_{wL}\frac{S_L}{A_L} - \tau_{wg}\left[\left(\frac{S_g}{A_g}\right) + \left(\frac{\tau_i}{\tau_{wg}}\right)\left(\frac{S_i}{A_L} + \frac{S_i}{A_g}\right)\right] +$$

$$(\rho_L - \rho_g)\, g \sin\alpha = 0 \qquad (8)$$

Applying constitutive equations and geometrical relationships, one can show that Eq. (8) is an implicit function of h_L/D. One problem encountered in solving Eq. (8) is the multiple roots which occur in some cases (Baker *et al.* 1988 and Crowley & Rothe 1988). Commonly, it is presumed that the smallest value is the physical one.

After solving this equation for h_L/D, the liquid holdup can be derived from a geometrical relationship:

$$E_L = \frac{\theta - \sin\theta}{2\pi} \qquad (9)$$

where

$$\theta = 2\cos^{-1}\left(1 - 2\frac{h_L}{D}\right) \qquad (10)$$

With the solved liquid holdup, Eq. (6) or (7) can be used to calculate the pressure gradient. Another way is to apply both equations by eliminating interfacial shear, i.e.,

$$-\left(\frac{dp}{dx}\right) = \frac{\tau_{wL}\, S_L + \tau_{wg}\, S_g}{A} +$$

$$\left(\frac{A_L}{A}\rho_L + \frac{A_g}{A}\rho_g\right) g \sin\alpha \qquad (11)$$

Notice that the first term in the right hand side (RHS) of Eq. (11) represents the frictional pressure gradient, and the second term represents the gravitational pressure gradient. Obviously, the accelerational pressure gradient has been neglected.

Constitutive Equations

Shear stress. The shear stresses in liquid-wall, gas-wall and interface are evaluated through friction factors as:

$$\tau_{wL} = f_{wL} \frac{\rho_L v_L^2}{2} \quad \tau_{wg} = f_{wg} \frac{\rho_g v_g^2}{2} \quad \tau_i = f_i \frac{\rho_g v_g^2}{2} \text{..... (12)}$$

where f_{wL} and f_{wg} are obtained as follows.

$$f = \frac{16}{Re} \quad \text{for } Re \leq 2000 \text{ .... (13)}$$

$$\frac{1}{\sqrt{f}} = 3.48 - 4 \log \left(\frac{2\varepsilon}{D} + \frac{9.35}{Re \sqrt{f}} \right)$$

for $Re > 2000$ .... (14)

Where ε is the pipe wall absolute roughness. Liquid and gas Reynolds numbers are defined as $Re_L = \rho_L v_L D_L / \mu_L$ and $Re_g = \rho_g v_g D_g / \mu_g$, with hydraulic diameters, D_L and D_g, given by Eq. (15):

$$D_L = \frac{4 A_L}{S_L} \qquad D_g = \frac{4 A_g}{(S_g + S_i)} \text{........................ (15)}$$

Experiments by Kowalski (1987), Andritsos & Hanratty (1987) and Andreussi & Persen (1987) all show that the liquid-wall friction factor deviates from the friction factor of single-phase flow due to the presence of interfacial waves. However, it is generally agreed that using new correlations for f_L other than the conventional one does not improve the prediction significantly. In this study, the effect of f_L on the model performance will be investigated.

Interfacial friction factor. A closure relationship for the interfacial friction factor is needed to complete the stratified flow model. In the original Taitel & Dukler model this friction factor was assumed to be equal to the gas-wall friction factor. Underprediction of the pressure gradient due to this assumption was reported by many later investigators. Many studies have been focused on improving the interfacial friction modelling.

An extensive evaluation of available interfacial friction correlations reveals that current methods for prediction of the interfacial friction are far from being satisfactory. It is found out that the correlation developed by Andritsos & Hanratty (1987) works well for small diameter pipes but overpredicts the friction factor when applied to large diameter pipes. A modified Duns & Ros correlation (Brill & Beggs 1986) used by Baker *et al.* (1988), on the other hand, underpredicts friction factor for small diameter pipes and gives a correct trend for large diameter pipes. Therefore, it is recommended to use a combination of the Andritsos & Hanratty (1987) correlation ($D \leq 0.127$ m or 5 in) and the Baker *et al.* (1988) suggested correlation, i.e.,

for $D \leq 0.127$ m

If $v_{sg} \leq v_{sg,t}$, then:

$$\frac{f_i}{f_{wg}} = 1 \text{ .... (16)}$$

If $v_{sg} > v_{sg,t}$, then:

$$\frac{f_i}{f_{wg}} = 1 + 15 \sqrt{\frac{h_L}{D}} \left(\frac{v_{sg}}{v_{sg,t}} - 1 \right) \text{................. (17)}$$

where $v_{sg,t}$ is the critical superficial gas velocity for the transition to wavy regime. From Andritsos & Hanratty (1987), this velocity can be approximated by:

$$v_{sg,t} = 5 \sqrt{\frac{101325}{p}} \text{ .... (18)}$$

where p is the pressure in Pa (N/m^2).

for $D > 0.127$ m

for $N_{we} N_\mu \leq 0.005$,

$$\varepsilon_i = \frac{34\,\sigma}{\rho_g v_L^2} \text{ .... (19)}$$

for $N_{we} N_\mu > 0.005$,

$$\varepsilon_i = \frac{170\,\sigma \left(N_{we} N_\mu\right)^{0.3}}{\rho_g v_L^2} \text{ .... (20)}$$

where ε_i is the interface absolute roughness. Baker *et al.* (1988) proposed that ε_i should be bounded between the pipe wall absolute roughness and $0.25(h_L/D)$. The Weber number, N_{we}, and the liquid viscosity number, N_μ, are defined as:

$$N_{we} = \frac{\rho_g v_L^2 \varepsilon_i}{\sigma} \text{ .... (21)}$$

$$N_\mu = \frac{\mu_L^2}{\rho_L \sigma \varepsilon_i} \text{ .... (22)}$$

Baker *et al.* (1988) suggested to replace the superficial gas velocity in the original Duns and Ros correlation with the interfacial velocity, v_i. In this study, v_L is substituted for v_i. From ε_i and Re_g, the interfacial friction factor is calculated from Eq. (14).

For flow pattern prediction, unfortunately, use of the above recommended correlations or any other correlations is questionable. This is due to the fact that available correlations were usually developed for actual stratified flow with low h_L/D, while for equilibrium stratified flow h_L/D can range from 0.0 to 1.0, theoretically. At present, a value of f_i = 0.0142 is used for equilibrium stratified flow in flow pattern determination. This constant value is suggested by Shoham & Taitel (1984).

INTERMITTENT FLOW MODEL

Intermittent flow is characterized by alternate flow of liquid and gas (Fig. 4). Plugs or slugs of liquid, which fill the entire pipe cross sectional area, are separated by gas pockets, which contain a stratified liquid layer flowing along the bottom of the pipe. The mechanism of the flow is that of a fast moving liquid slug overriding the slow moving liquid film ahead of it. The liquid in the slug body may be aerated by small bubbles which are concentrated towards the front of the slug and at the top of the pipe.

Intermittent flow has been studied by many investigators. Recently, a consistent approach has been carried out by Taitel & Barnea (1990). They presented a general approach to determine the hydrodynamics of the liquid film of a slug unit using a very detailed one-dimensional channel flow model. The disadvantage of this general approach is the requirement of numerical integration. For practical application, a model which assumes a uniform liquid level in the film zone is believed to be sufficient.

With this assumption and considering incompressible liquid and gas phases, an overall liquid mass balance over a slug unit gives:

$$v_{sL} L_u = v_L E_s L_s + v_f E_f L_f \quad \text{.......................} (23)$$

where, E_s and E_f are the liquid holdups in slug body and film zone, respectively. A mass balance can be also applied at two cross sections relative to a coordinate system moving at the translational velocity. For the liquid phase, this results in

$$(v_t - v_L) E_s = (v_t - v_f) E_f \quad \text{.............................} (24)$$

The total volumetric flow rate is constant at any cross section in a slug unit. For the slug body and the film zone cross sections, this implies:

$$v_s = v_{sL} + v_{sg} = v_L E_s + v_b (1 - E_s) \quad \text{............} (25)$$

$$v_s = v_f E_f + v_g (1 - E_f) \quad \text{..............................} (26)$$

where v_s represents the mixture velocity in the slug body.

The above four equations yield several important relationships. From Eq. (25), the liquid velocity in the slug body, v_L, is obtained. Then, Eq. (24) is rearranged to give an expression for the liquid velocity in the film zone, v_f. Using Eq. (26), an expression for the gas velocity in the film zone, v_g, can be obtained. The average liquid holdup for a slug unit, E_L, is defined as:

$$E_L = \frac{E_s L_s + E_f L_f}{L_u} \quad \text{....................................} (27)$$

From Eqs. (23), (24) and (25), a relationship for E_L can be derived:

$$E_L = \frac{v_t E_s + v_b (1 - E_s) - v_{sg}}{v_t} \quad \text{.....................} (28)$$

Since we consider a uniform liquid level along the film zone, a combined momentum equation, similar to Eq. (8) in stratified flow, can be obtained for the film zone:

$$\tau_f \frac{S_f}{A_f} - \tau_g \left[\left(\frac{S_g}{A_g}\right) + \left(\frac{\tau_i}{\tau_g}\right)\left(\frac{S_i}{A_f} + \frac{S_i}{A_g}\right)\right] +$$

$$(\rho_L - \rho_g) g \sin \alpha = 0 \quad \text{.................................} (29)$$

Analogously, Eq. (29) is solved for the equilibrium liquid level, or the liquid holdup in the film zone, E_f. Then, the liquid and gas velocities and the shear stresses can be evaluated. Considering the slug length to be known, the slug unit length can be obtained from Eq. (23) and $L_u = L_s + L_f$:

$$L_u = L_s \frac{v_L E_s - v_f E_f}{v_{sL} - v_f E_f} \quad \text{...................................} (30)$$

The average pressure gradient for intermittent flow is calculated by using a force balance over a slug unit:

$$-\left(\frac{dp}{dx}\right) = \rho_u g \sin \alpha +$$

$$\frac{1}{L_u}\left[\left(\frac{\tau_s \pi D}{A} L_s\right) + \left(\frac{\tau_f S_f + \tau_g S_g}{A} L_f\right)\right] \quad \text{.............} (31)$$

where ρ_u is the average fluid density of a slug unit:

$$\rho_u = E_L \rho_L + (1 - E_L) \rho_g \quad \text{.............................} (32)$$

The first term of the RHS of Eq. (31) is the gravitational pressure gradient whereas the second term is the frictional pressure gradient, which results from the friction loss in the slug body as well as the friction loss in the film zone.

Constitutive Equations

Shear stress. The shear stresses appearing in Eq. (29) are calculated in a similar manner as in stratified flow, i.e.,

$$\tau_f = f_f \frac{\rho_L |v_f| v_f}{2} \qquad \tau_g = f_g \frac{\rho_g |v_g| v_g}{2}$$

$$\tau_i = f_i \frac{\rho_g |v_g - v_f| (v_g - v_f)}{2} \quad \text{.......................... (33)}$$

where f_f and f_g are evaluated using Eq. (13) or (14) with $Re_f = \rho_L v_f D_L/\mu_L$ and $Re_g = \rho_g v_g D_g/\mu_g$. Hydraulic diameters are defined exactly as in stratified flow. A constant value of f_i = 0.0142 is used for the interfacial friction factor.

The shear stress in the slug body, τ_s, is calculated as:

$$\tau_s = f_s \frac{\rho_s v_s^2}{2} \quad \text{.. (34)}$$

where f_s is obtained from Eq. (13) or (14) using $Re_s = \rho_s v_s D/\mu_s$. ρ_s and μ_s are the mixture density and viscosity in the slug body, respectively:

$$\rho_s = E_s \rho_L + (1 - E_s) \rho_g \quad \text{.............................. (35)}$$

$$\mu_s = E_s \mu_L + (1 - E_s) \mu_g \quad \text{.............................. (36)}$$

Correlations for v_t and v_b. The correlation for elongated (Taylor) bubble translational velocity is based on Bendiksen's recommendation (Bendiksen 1984):

$$v_t = C v_s + 0.35 \sqrt{g D} \sin \alpha +$$

$$0.54 \sqrt{gD} \cos \alpha \quad \text{...................................... (37)}$$

where the value of C depends on the liquid velocity profile in the slug body. C = 1.2 is used for turbulent flow and C = 2 is used for laminar flow.

The velocity of dispersed bubbles in the slug body is given by:

$$v_b = 1.2\, v_s +$$

$$1.53 \left[\frac{\sigma g (\rho_L - \rho_g)}{\rho_L^2}\right]^{1/4} E_s^{0.1} \sin \alpha \quad \text{.............. (38)}$$

where $E_s^{0.1}$ is included to account the effect of "bubble swarm" in the slug body (Ansari 1988).

Liquid holdup in slug Body. The correlation developed for liquid holdup in the slug body by Gregory *et al.* (1978), given below, is used in this study.

$$E_s = \frac{1}{1 + \left(\frac{v_s}{8.66}\right)^{1.39}} \quad \text{.................................. (39)}$$

The calculated E_s is bounded between 1.0 and 0.48.

Slug length. For slug length, we use the correlation developed by Scott (1987):

$$\text{Ln}(L_s) = -26.6 +$$

$$28.5 \left[\text{Ln}(D) + 3.67\right]^{0.1} \quad \text{............................ (40)}$$

If D < 0.0381 mm (1.5 in), an approximate value of L_s = 30 D is used.

ANNULAR FLOW MODEL

The liquid phase in annular flow exists in two forms: a liquid film flowing along the pipe wall; and, liquid droplets entrained in the gas core (see Fig. 5). Unlike the vertical flow case, the liquid film in the horizontal and inclined configurations is not circumferentially uniform, but is usually thicker at the bottom than at the top of the pipe.

Early studies for annular flow were summarized by Hewitt & Hall-Taylor (1970). The classical treatment for annular flow has been the use of the well-known triangular relationship between the film flow rate, the film thickness and the pressure gradient. This treatment ignores the liquid secondary flow effects, circumferential variations of the film thickness, and the deposition and entrainment rates. These phenomena are important for horizontal and inclined annular flow. Therefore, two-dimensional models are proposed to incorporate these mechanisms (James *et al.* 1987 and Laurinat *et al.* 1985). Nevertheless, in these models, complex mathematical formulations are involved, and numerical methods are often required for the solution. For vertical annular flow, on the other hand, the one-dimensional two-fluid approach has used by Oliemans *et al.* (1986) and later by Alves *et al.* (1988). Comparing with field data, Ansari (1988) shows that this approach gives excellent results.

In the present work, the two-fluid approach is extended to fully developed steady state annular flow in pipelines. For simplicity, an average film thickness is assumed. In the gas core, the droplets are assumed to travel at the same velocity as the gas phase. Thus, the gas core can be treated as a homogeneous fluid. Because of these assumptions, the treatment of annular flow is similar to stratified flow, but with a different geometrical configuration. Here, the two fluids are the liquid film and the gas core which includes the gas and the entrained liquid droplets.

Momentum balances on the liquid film and the gas core yield

$$- A_f \left(\frac{dp}{dx}\right) + \tau_i S_i - \tau_{wL} S_L - A_f \rho_L g \sin\alpha = 0 \quad \text{......(41)}$$

$$- A_c \left(\frac{dp}{dx}\right) - \tau_i S_i - A_c \rho_c g \sin\alpha = 0 \quad \text{......(42)}$$

where ρ_c is the mixture density in the gas core and is given by:

$$\rho_c = E_c \rho_L + \left(1 - E_c\right) \rho_g \quad \text{......(43)}$$

The liquid holdup in the gas core is related to liquid entrainment fraction, FE, as follows:

$$E_c = \frac{v_{sL} FE}{v_{sg} + v_{sL} FE} \quad \text{......(44)}$$

Eliminating the pressure gradient from these equations gives the combined momentum equation:

$$\tau_{wL} \frac{S_L}{A_f} - \tau_i S_i \left(\frac{1}{A_f} + \frac{1}{A_c}\right) + (\rho_L - \rho_c) g \sin\alpha = 0 \quad \text{......(45)}$$

Similar to the stratified flow case, all the geometric parameters in Eq. (45) are functions of δ/D, the dimensionless average film thickness. Thus, the combined momentum equation can be solved for this unknown, from which the liquid holdup can be calculated:

$$E_L = 1 - \left(1 - 2\frac{\delta}{D}\right)^2 \frac{v_{sg}}{v_{sg} + v_{sL} FE} \quad \text{......(46)}$$

The pressure gradient can be evaluated using Eqs. (41) and (42):

$$- \left(\frac{dp}{dx}\right) = \frac{\tau_{wL} S_L}{A} + \left(\frac{A_f}{A} \rho_L + \frac{A_c}{A} \rho_c\right) g \sin\alpha \quad \text{......(47)}$$

Clearly, the total pressure gradient is a summation of the frictional pressure gradient (the first term of the RHS), and the gravitational pressure gradient (the second term of the RHS). Again, the accelerational pressure gradient is neglected.

Constitutive Equations

Shear stress. The shear stresses are defined as follows:

$$\tau_{wL} = f_f \frac{\rho_L v_f^2}{2} \qquad \tau_i = f_i \frac{\rho_c (v_c - v_f)^2}{2} \quad \text{......(48)}$$

where f_f is calculated from Eq. (13) or (14) using $Re_L = \rho_L v_f D_L/\mu_L$, with the hydraulic diameter defined as $D_L = 4\delta(D-\delta)/D$.

Using an overall liquid volumetric flow rate balance for the film leads to the following relationship for the liquid film velocity, v_f:

$$v_f = \frac{v_{sL}\left(1 - FE\right)}{4 \frac{\delta}{D}\left(1 - \frac{\delta}{D}\right)} \quad \text{......(49)}$$

Similarly, for the gas core, the mixture velocity is given by:

$$v_c = \frac{v_{sg} + v_{sL} FE}{\left(1 - 2\frac{\delta}{D}\right)^2} \quad \text{......(50)}$$

Liquid entrainment and interfacial friction factor. To complete the annular flow model, closure relationships for the interfacial friction factor and the liquid entrainment fraction are needed. Only few correlations have been developed from experimental data for horizontal annular flow (Henstock & Hanratty (1976), Laurinat *et al.* (1984)). No data are available for inclined annular flow. Consequently, correlations developed for vertical annular flow are also considered in this study (Wallis (1969), Whalley & Hewitt (1978) and Oliemans *et al.* (1986)). It is found out that the combination of the liquid entrainment and interfacial friction correlations proposed by Oliemans *et al.* (1986) gives the best results. These correlations are given as follows

$$\frac{FE}{1 - FE} =$$

$$10^{\beta_o}\, \rho_L^{\beta_1}\, \rho_g^{\beta_2}\, \mu_L^{\beta_3}\, \mu_g^{\beta_4}\, \sigma^{\beta_5}\, D^{\beta_6}\, v_{sL}^{\beta_7}\, v_{sg}^{\beta_8}\, g^{\beta_9} \quad (51)$$

$$f_i = f_c \left[1 + 2250 \frac{\left(\frac{\delta}{D}\right)}{\left(\frac{\rho_c (v_c - v_f)^2\, \delta}{\sigma}\right)} \right] \quad (52)$$

where the β parameters are regression coefficients. Eq. (52) is a modification of the original correlation carried out by Crowley & Rothe (1986). The core friction factor, f_c, can be calculated from Eq. (13) or (14) using the following definition of the Reynolds number :

$$Re_c = \frac{\rho_c\, v_c\, D_c}{\mu_c} \quad (53)$$

where

$$\mu_c = E_c\, \mu_L + \left(1 - E_c\right) \mu_g \quad (54)$$

$$D_c = D - 2\,\delta \quad (55)$$

DISPERSED BUBBLE FLOW MODEL

Among the four flow patterns, the model for dispersed bubble flow is the simplest one. Due to no slippage between the phases, the pseudo-single phase model with average properties is suitable for this flow pattern. The liquid holdup is thus the no-slip liquid holdup:

$$E_L = \frac{v_{sL}}{v_m} \quad (56)$$

Calculation of the pressure gradient calculation can be carried out as in single phase flow with average mixture density and velocity:

$$-\left(\frac{dp}{dx}\right) = \frac{2\, f_m\, \rho_m\, v_m^2}{D} + \rho_m\, g \sin\alpha \quad (57)$$

EVALUATION

PIPELINE DATA BASE

The applicability of the proposed comprehensive mechanistic model is assessed through comparisons with actual data. For this purpose, a pipeline data bank has been established. This data base contains a total of 426 field and laboratory data from various sources, as shown in Table 1.

The 1988 version of the A. G. A. gas-liquid pipeline data base contains 455 data points (Crowley 1988). Theses data are from measurements in a wide variety of gas and oil pipelines. Thus, it provides an appropriate source of data for statistical analysis. However, many data points contained in this data base are either almost identical or not reliable. As an example, there are cases where unrealistically low pressure drops were reported for very long pipelines. Data of this nature are discarded. Another concern in the evaluation process is the accuracy of the fluid physical property prediction. To reflect the performance of the model, errors from fluid physical property calculation should be minimized. For a compositional system containing free water a possible water-oil emulsion may occur. Therefore, compositional data containing free water are not considered. As a consequent of all these considerations, only 79 data points are selected for this study, among which 25 data are from compositional systems.

Field measurements by Mcleod *et al.* (1971) are also included in our data base. These are high quality data taken in an offshore pipeline of 152.4-mm diameter. The fluids are modelled as a black-oil system.

Additional laboratory data from Eaton & Brown (1965) and Payne *et al.* (1979) are included. Although these data were obtained in small diameter pipes, the operational pressures are very close to field conditions.

STATISTICAL PARAMETERS

The statistical parameters used in this study are defined in Table 2 and are explained below:

The average percentage error, ε_1, and the average error, ε_4, are measures of the agreement between predicted and measured data. They indicate the degree of overprediction (positive values) or underprediction (negative values). The absolute average percentage error, ε_2, and the absolute average error, ε_5, are considered to be more important than ε_1 and ε_4, because the negative and the positive errors do not cancel out. The standard deviations, ε_3 and ε_6, indicate the scatter of the errors with respect to their corresponding average errors, ε_1 and ε_4.

The first three parameters are more appropriate to be used for the evaluation of small values, whereas the rest three are better for large values. In this study, all the six parameters are considered in the evaluation

RESULTS AND DISCUSSIONS

The evaluation in this study is only carried out for pressure drops, since most of the cases in the data base do not contain liquid holdup values. The commonly used correlations of Beggs and Brill, Mukherjee and Brill, Dukler and Dukler with the Eaton holdup correlation (Brill & Beggs 1986) have also been included in the evaluation for the purpose of comparison.

The overall evaluation of the comprehensive mechanistic model using the entire data base is shown in Table 3. The calculated and measured pressure drops are also plotted to give an overall picture of the performance of the model (Fig. 6). The model has negative values for ε_1 and ε_4, indicating its underprediction for pressure drops. All the other statistical parameters of the model are the smallest, which demonstrates its superior performance over all the correlations. Of all 426 cases, there is only one case where the model has a convergent problem, whereas all correlations have more than five troublesome cases. In this respect, the comprehensive mechanistic model is also the best.

For a flow pattern dependent model such as the comprehensive model, the evaluation should be also carried out for each of the individual flow pattern models. Here, the entire data base is separated into groups in which all cases have the same dominant flow pattern (>75% of the total pipe length), namely stratified, intermittent, annular flow and dispersed bubble flow. Then, a separate evaluation is conducted for each flow pattern. The results can be found in Table 4-6 and Figure 7-9. It can be seen that all these models, particularly the intermittent flow model, perform better than any of the correlations. No evaluation can be done for the dispersed bubble flow model because there is no dispersed bubble flow dominated cases.

The degree of uncertainty in the calculation of the liquid-wall friction factor for stratified flow has also been studied. A sensitivity study is undertaken by varying the value of f_L $\pm 25\%$ around its calculated value. The results are reported in Table 7. As shown, except for some changes in ε_1 and ε_4, the other parameters remain almost the same. This suggests that the performance of the stratified flow model is generally not sensitive to f_L.

CONCLUSIONS AND RECOMMENDATIONS

Based on the results of this study, the following conclusions have been reached:

1. A comprehensive mechanistic model, which is capable of predicting two-phase flow pattern, liquid holdup and pressure drop, has been formulated.

2. The consistency and applicability of the comprehensive mechanistic model have been demonstrated by its overall superior performance over any of the compared correlations for the wide variety of data contained in the data base.

3. All individual flow pattern models give better results than any of the empirical correlations.

For future studies, the following recommendations are made:

1. The major uncertainty for the stratified flow model is the interfacial friction factor. Future studies should be focused on improving our understanding of the interfacial shear phenomena, and developing more accurate predictive methods.

2. For annular flow, the correlations for liquid entrainment and interfacial friction factor are all developed from vertical annular flow experiments. More studies are needed for horizontal and inclined annular flow.

3. Small diameter laboratory data represent a large portion of the data base used in this study. More high quality field data are needed to further verify the mechanistic model.

ACKNOWLEDGMENT

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NOMENCLATURE

A	=	pipe cross sectional area or area occupied by fluid
C	=	constant coefficient
dA_L/dh_L	=	differentiation of A_L with respect to h_L
dp/dx	=	pressure gradient
D	=	pipe diameter or hydraulic diameter
E	=	liquid holdup
f	=	fanning friction factor
FE	=	liquid entrainment fraction
g	=	acceleration of gravity
h	=	liquid level
L	=	length
N	=	number of points
N_{we}	=	Weber number
N_μ	=	liquid viscosity number
p	=	pressure
Re	=	Reynolds number
s	=	wetted periphery or sheltering coefficient
v	=	velocity

Greek Letters

α = pipe inclination angle, positive for upward
β = regression coefficient
δ = film thickness
Δp = pressure drop
ε = roughness or error parameter
θ = angle subtended by interface
μ = viscosity
ρ = density
σ = surface tension
Σ = summation
τ = shear stress

Subscripts

b = bubble
c = core or calculated
f = film
g = gas phase
i = interface
L = liquid phase
m = measured or mixture
s = superficial or slug
t = transition or translational
u = slug unit
w = wall

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Table 1: Pipeline Data Base

	Data Sources	Nominal Pipe Diameter (mm)	No. of Data Points	Fluid System
Field Data	A. G. A.	76.2 - 660.4	79	54-Black Oil System 25-Compositional System
	Mcleod *et al.*	152.4	12	Black Oil System
Lab. Data	Eaton *et al.*	50.8 101.6	139 97	Natural Gas / Water, Crude or Distillate
	Payne *et al.*	50.8	99	Natural Gas / Water
Total No. of Data Points			426	

Table 2

Definitions of Statistical Parameters

Statistical Parameters	Definitions	Unit
ε_1	$\frac{1}{N}\left[\Sigma\left(\frac{\Delta p_c-\Delta p_m}{\Delta p_m}\times 100\right)\right]$	%
ε_2	$\frac{1}{N}\left(\Sigma\left\lvert\frac{\Delta p_c-\Delta p_m}{\Delta p_m}\times 100\right\rvert\right)$	%
ε_3	$\sqrt{\frac{\Sigma\left(\frac{\Delta p_c-\Delta p_m}{\Delta p_m}\times 100-\varepsilon_1\right)^2}{N-1}}$	%
ε_4	$\frac{1}{N}[\Sigma(\Delta p_c-\Delta p_m)]$	$\times 10^4$ Pa
ε_5	$\frac{1}{N}(\Sigma\lvert\Delta p_c-\Delta p_m\rvert)$	$\times 10^4$ Pa
ε_6	$\sqrt{\frac{\Sigma[(\Delta p_c-\Delta p_m)-\varepsilon_4]^2}{N-1}}$	$\times 10^4$ Pa

Notes:

Δp_c = Calculated Pressure Drop ($\times 10^4$ Pa)
Δp_m = Measured Pressure Drop ($\times 10^4$ Pa)

Table 3

Overall Evaluation of the Comprehensive Mechanistic Model Using Entire Data Base

No.	Model or Correlation	No. of Data Points	Statistical Parameters					
			ε_1 (%)	ε_2 (%)	ε_3 (%)	ε_4 $\times 10^4$ (Pa)	ε_5 $\times 10^4$ (Pa)	ε_6 $\times 10^4$ (Pa)
1	**This Model**	425	-11.7	30.5	50.6	-8.6	12.2	22.0
2	Beggs & Brill	415	10.9	35.0	94.2	9.6	13.2	31.6
3	Muk. & Brill	411	39.4	60.5	128.	16.3	21.3	40.5
4	Dukler	415	32.9	43.0	107.	17.3	18.7	36.7
5	Dukler-Eaton	419	21.5	35.4	89.3	13.8	16.0	33.9

Table 4

Evaluation of Stratified Flow Model Using Cases with 75% Stratified Flow

No.	Model or Correlation	No. of Data Points	Statistical Parameters					
			ε_1 (%)	ε_2 (%)	ε_3 (%)	ε_4 $\times 10^4$ (Pa)	ε_5 $\times 10^4$ (Pa)	ε_6 $\times 10^4$ (Pa)
1	**This Model**	89	-18.0	34.6	49.1	-3.2	8.1	14.3
2	Beggs & Brill	86	-9.0	34.9	53.7	6.6	9.0	16.2
3	Muk. & Brill	83	16.7	66.9	79.0	16.0	19.3	19.4
4	Dukler	86	32.1	59.2	80.8	21.1	22.5	25.1
5	Dukler-Eaton	86	23.1	54.4	78.8	17.4	19.0	23.1

Table 5

Evaluation of Intermittent Flow Model Using Cases with 75% Intermittent Flow

No.	Model or Correlation	No. of Data Points	Statistical Parameters					
			ε_1 (%)	ε_2 (%)	ε_3 (%)	ε_4 $\times 10^4$ (Pa)	ε_5 $\times 10^4$ (Pa)	ε_6 $\times 10^4$ (Pa)
1	**This Model**	121	-18.9	22.7	19.8	-8.7	9.3	12.6
2	Beggs & Brill	129	23.3	31.6	33.0	7.9	9.6	13.9
3	Muk. & Brill	127	27.1	40.0	54.7	4.8	10.1	15.7
4	Dukler	128	28.5	34.4	35.5	10.5	11.4	16.5
5	Dukler-Eaton	129	22.5	29.9	32.4	9.6	10.7	18.2

Table 6

Evaluation of Annular Flow Model Using Cases with 75% Annular Flow

No.	Model or Correlation	No. of Data Points	Statistical Parameters					
			ε_1 (%)	ε_2 (%)	ε_3 (%)	ε_4 $\times 10^4$ (Pa)	ε_5 $\times 10^4$ (Pa)	ε_6 $\times 10^4$ (Pa)
1	**This Model**	123	-3.6	39.2	76.5	-17.9	24.0	33.7
2	Beggs & Brill	114	33.3	41.7	163.	21.9	24.7	50.2
3	Muk. & Brill	117	83.0	94.1	215.	35.4	43.3	63.3
4	Dukler	114	49.8	56.5	187	27.8	30.4	53.7
5	Dukler-Eaton	119	29.8	41.8	149	22.1	27.3	54.2

Table 7

Sensitivity Analysis of Liquid-Wall Friction Factor on the Performance of Stratified Flow Model

No.	Friction Factor f_L	No. of Data Points	Statistical Parameters					
			ε_1 (%)	ε_2 (%)	ε_3 (%)	ε_4 $\times 10^4$ (Pa)	ε_5 $\times 10^4$ (Pa)	ε_6 $\times 10^4$ (Pa)
1	0.75 f_L	89	-22.1	35.9	48.4	-4.7	8.5	14.4
2	f_L	89	-18.0	34.6	49.1	-3.2	8.1	14.3
3	1.25 f_L	89	-14.4	34.4	49.8	-1.9	8.0	14.3

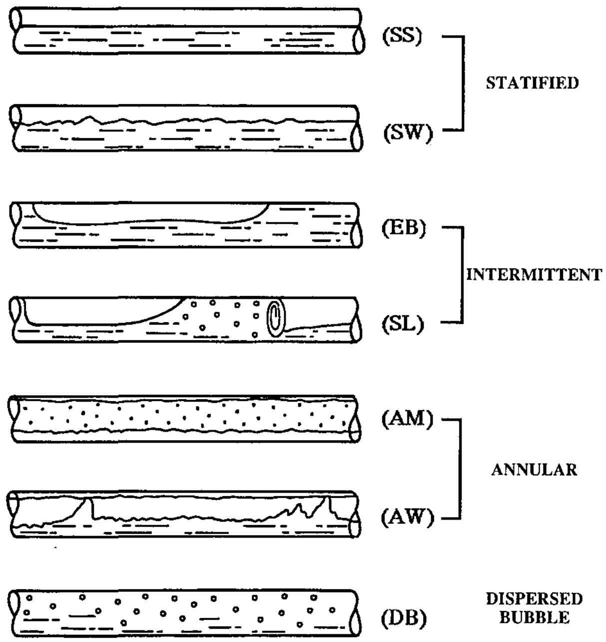

Figure 1 - Flow Patterns in Horizontal and Near Horizontal Pipes

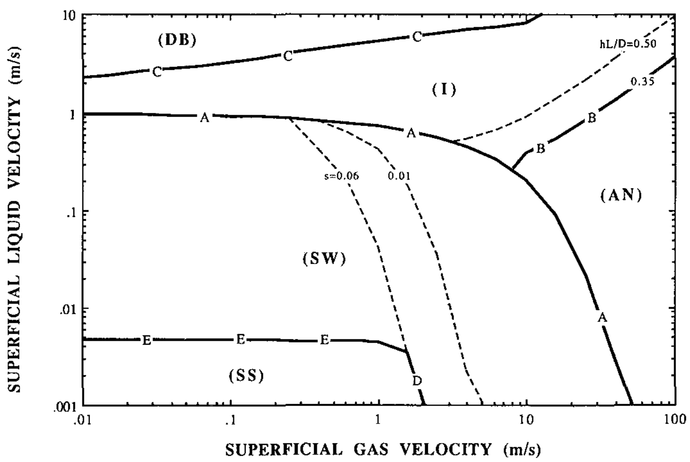

Figure 2 - Flow Pattern Map (Air-Water in 5-cm Pipe of -1 Degree Inclination)

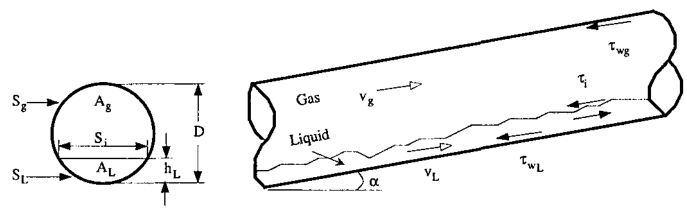

Figure 3 - Physical Model for Stratified Flow

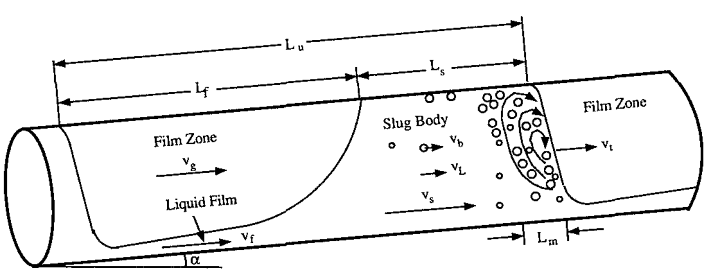

Figure 4 - Physical Model For Intermittent Flow

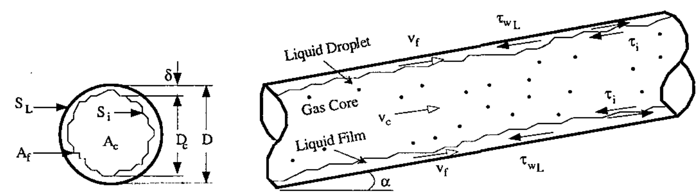

Figure 5 - Physical Model for Annular Flow

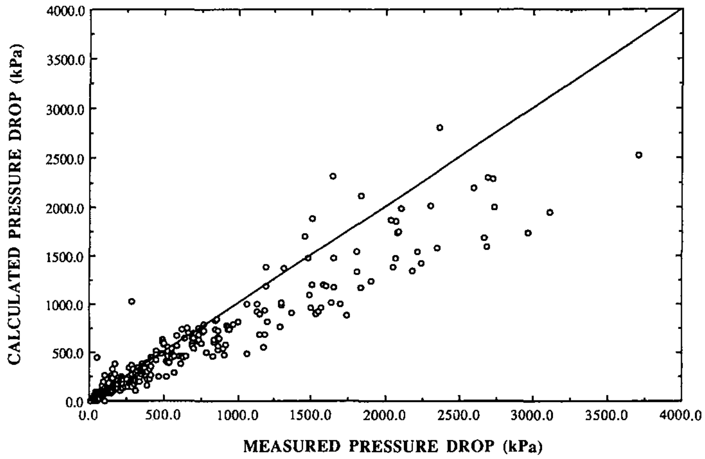

Figure 6 - Performance of the Comprehensive Model Using Entire Data Bank

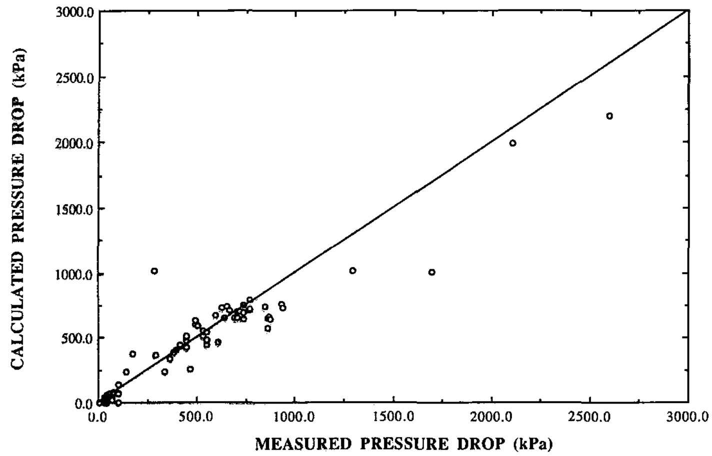

Figure 7 - Performance of Stratified Flow Model Using Cases with 75% Stratified Flow

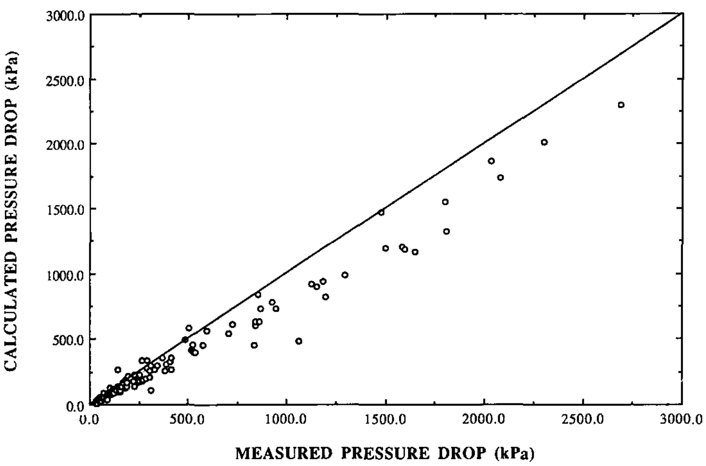

Figure 8 - Performance of Intermittent Flow Model Using Cases with 75% Intermittent Flow

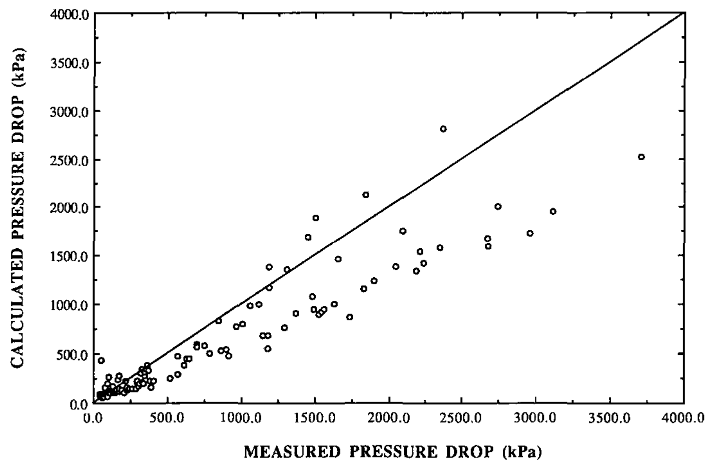

Figure 9 - Performance of Annular Flow Model Using Cases with 75% Annular Flow